

1.0 Anthropometry

- Major branch of anthropology
- Studies physical measurement of the human body to determine differences in individuals and groups
- A wide variety of physical measurements are required to describe and differentiate the characteristics of race, sex, age and body type.

Past emphasis of anthropometry was on evolution and history.

Technology has driven more recent studies, for example:

- Man-machine interfaces
- Workplace design
- Cockpits
- Pressure suits
- Armour

These studies are usually satisfied by basic linear, area and volume measures. However, human movement analysis also requires kinetic measures:

- Masses, moments of inertia and their locations
- Joint centers of rotation

- Origin and insertion of muscles
- Angles of pull of tendons
- Length and cross-sectional area of muscles

1.0.1 Segment Dimensions

- Most basic body dimension is the length of segments between each joint.

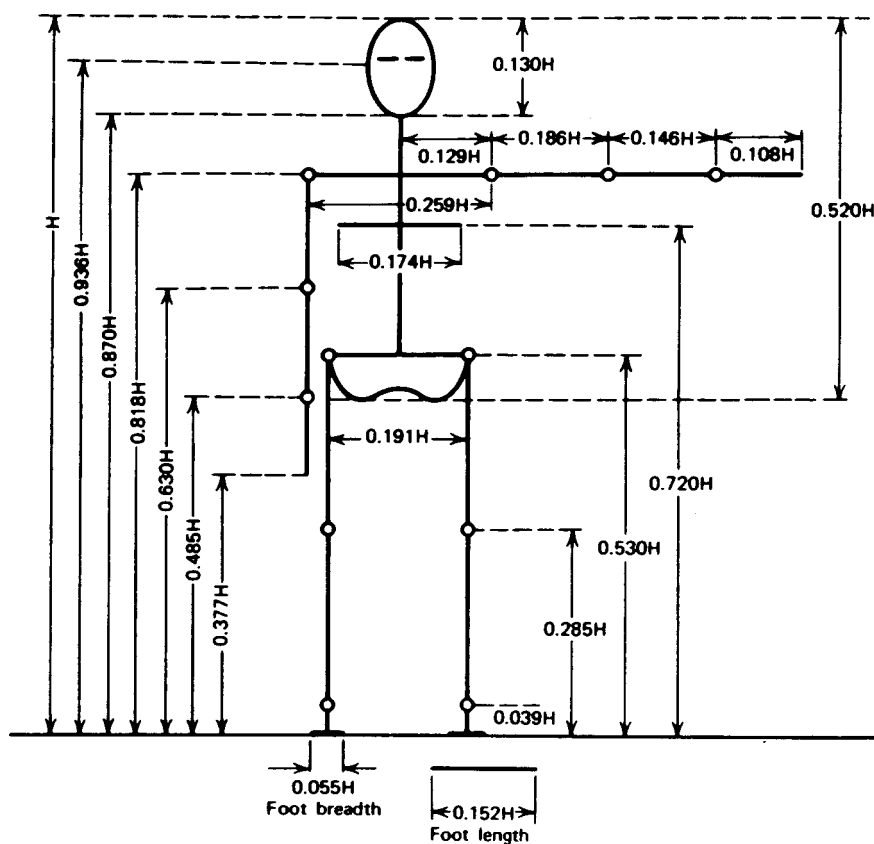


Figure 3.1 Body segment lengths expressed as a fraction of body height H .

- Varies with body build, sex, race and age.

Kinematic and kinetic analysis requires data regarding mass distribution, moments of inertia and the like.

1.1.1 Whole-body density

The body consists of many types of tissue – each with its own density.

For example:

Cortical bone $\rho \sim 1.8$

Muscle tissue $\rho \geq 1.0$

Fat $\rho < 1.0$

Lung $\rho < 1.0$

Average body density is a function of body build or somatotype:

Endomorph – short and fat

Mesomorph – stout and muscular

Ectomorph – tall and thin.

Usually no definite delineations exist among the types.

Most people have a somatotype consisting of components of all three types.

Modern ratings (like Heath-Carter somatotype method) are a qualitative assessment of the amount of all three components:

- Endomorphic component \sim relative fatness of the body
- Mesomorphic component \sim musculoskeletal rating per unit of body height.

- Ectomorphic component ~ “linearity” of the body.

Drills and Contini (1966) developed an expression for body density d as a function of ponderal index c :

$$c = \frac{h}{w^{1/3}}$$

Where:

w is in pounds

h is in inches

$$\left[\begin{aligned} c &= \frac{39.4}{(2.2)^{1/3}} \frac{h}{w} && w \text{ in } Kg \text{ and } h \text{ in } m. \\ &= 30.29 \frac{h}{w^{1/3}} && \text{inch } lb^{-\frac{1}{3}} \end{aligned} \right]$$

$$d = 0.69 + 0.0297c \quad Kg / L$$

In metric units – h is in meters and w is in kilograms

$$d = 0.69 + 0.9c \quad Kg / L$$

Example:

Calculate the whole-body density of an adult with

$$h = 5' 10'' \text{ and } w = 170lbs$$

Imperial:

$$c = \frac{h}{w^{1/3}} = \frac{70}{(170)^{1/3}} = 12.64 \text{ inch.lb}^{-1/3}$$

$$\begin{aligned} \therefore d &= 0.69 + 0.0297c \\ &= 0.69 + 0.297 \times 12.64 \\ &= 1.065 \text{ Kg/L} \end{aligned}$$

metric:

$$h = \frac{70}{39.4} = 1.78 \text{ m}$$

$$w = \frac{170}{2.2} = 77.3 \text{ Kg}$$

$$c = \frac{h}{w^{1/3}} = \frac{1.78}{(77.3)^{1/3}} = 0.418 \text{ m.Kg}^{-1/3}$$

$$d = 0.69 + 0.9c = 0.69 + 0.9 \times 0.418 = 1.066 \text{ Kg/L}$$

The ponderal index is a measure of stature or stoutness.

⇒ short, fat people have a lower ponderal index than tall thin people.

1.1.2 Segment density

- Each body segment has a unique combination of bone, muscle, fat and other tissue.
⇒ Density within a given segment is not uniform.
- Distal segments usually have higher proportion of bone
⇒ $\rho_{\text{distal segments}} > \rho_{\text{proximal segments}}$ (usually)
- Individual segments increase their density as average body density increases.

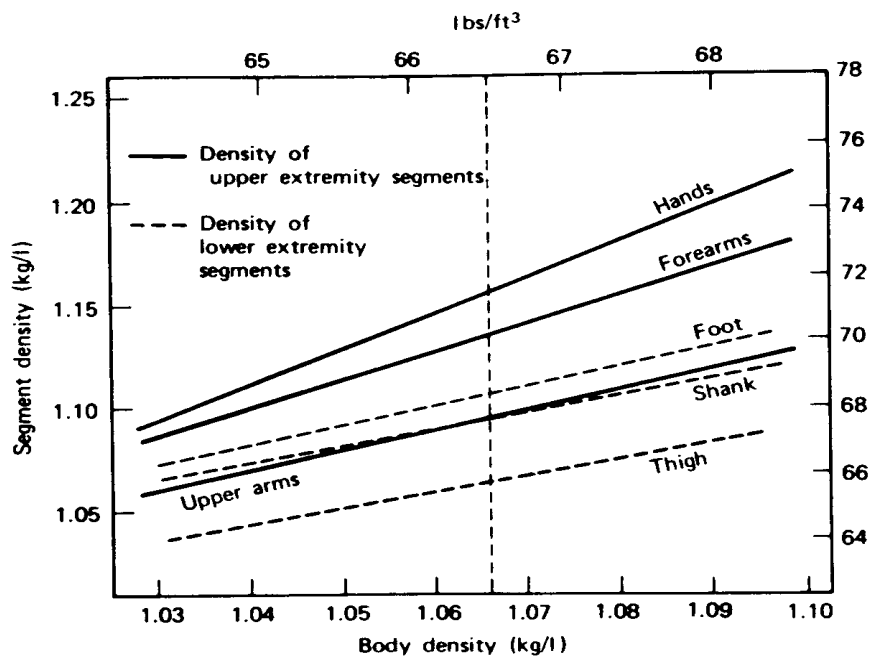


Figure 3.2 Density of limb segments as a function of average body density.

1.1.3 Segmental Mass and Centre of mass

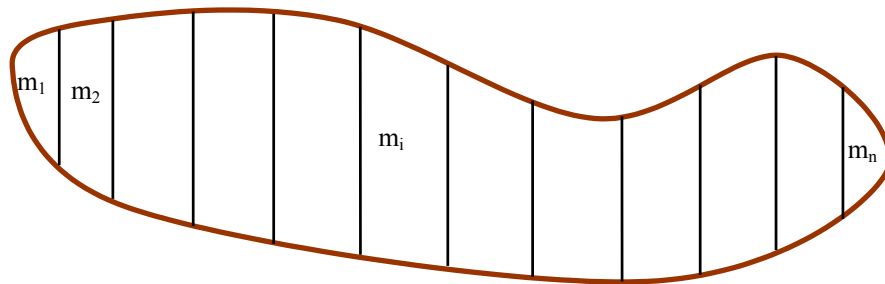
- Terms – centre of mass and center of gravity are often used interchangeably

Are they the same?

If over dimension of body $g = \text{const.}$

\Rightarrow centre of mass = centre of gravity

Consider a body segment of mass M –subdivided into n sections:



If $m_i =$ mass of the i^{th} section, then:

$$M = \sum_{i=1}^n m_i$$

If $\rho_i =$ density of the i^{th} section and $V_i =$ the corresponding volume, then,

$$m_i = \rho_i V_i$$

If the density is assumed uniform over the segment, ie,

$$\rho = \rho_1 = \rho_2 = \rho_3 = \dots = \rho_n$$

or

$$\rho = \rho_i \quad ; \quad i = 1, n$$

then

$$m_i = \rho V_i$$

and so

$$M = \rho \sum_{i=1}^n V_i$$

Suppose we measure distance (x) from one end of the segment. The centre of mass of the segment is such that:

$$Mx = \sum_{i=1}^n m_i x_i$$

[Gravitational torque of centre of mass about an axis
= Sum of the torques of each section about that axis.]

\Rightarrow

$$x = \frac{1}{M} \sum_{i=1}^n m_i x_i$$

In three dimensions we have,

$$\mathbf{r} = \frac{1}{M} \sum_{i=1}^n m_i \mathbf{r}_i$$

where

$$\mathbf{r}_i = (x_i, y_i, z_i)^T = \text{centre of mass of section } i.$$

and

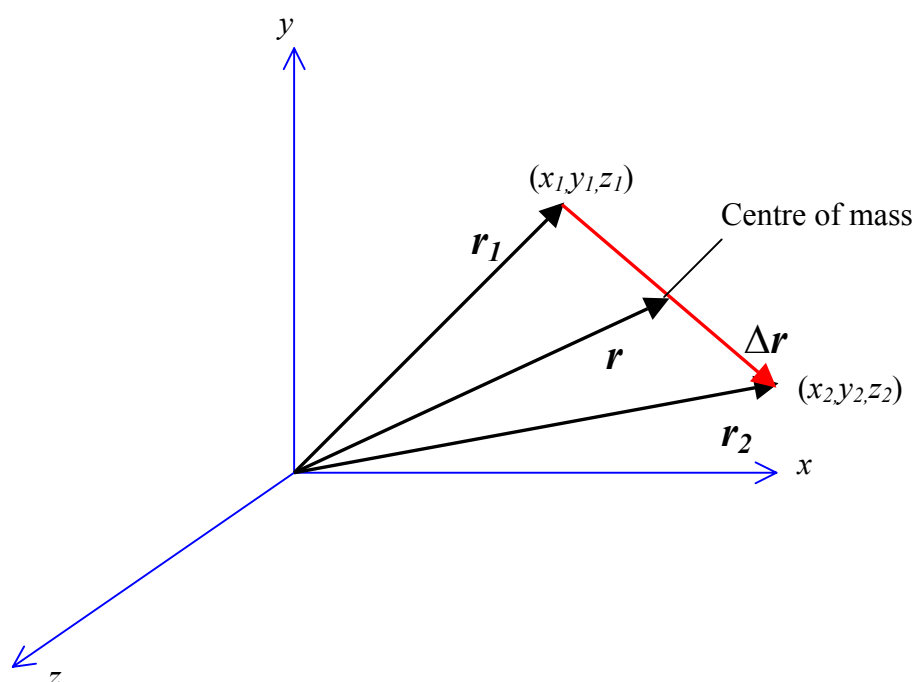
$$\mathbf{r} = (x, y, z)^T = \text{centre of mass of segment.}$$

We can now represent the complex distributed mass by a single mass M located at distance x from one end of the segment.

Often in anthropometric data, the centre of mass of a body segment is given as a fraction of the length of the segment measured from one end (See Table 3.1 of Winter).

The end points of the segments are referred to as markers.

Consider a body segment with markers at \mathbf{r}_1 and \mathbf{r}_2 :



If $\Delta \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$, then $\Delta r = \text{length of segment}$.

Suppose the centre of mass is located a fraction f of Δr from \mathbf{r}_1 . Then:

$$\begin{aligned}\mathbf{r} &= \mathbf{r}_1 + f \Delta \mathbf{r} \\ &= \mathbf{r}_1 + f(\mathbf{r}_2 - \mathbf{r}_1) \\ &= \mathbf{r}_1(1 - f) + f\mathbf{r}_2\end{aligned}$$

Exercise:

Express \mathbf{r} as measured from \mathbf{r}_2 .

$$\boxed{\mathbf{r} = \mathbf{r}_2 - (1 - f)\Delta \mathbf{r}}$$

Example:

From anthropometric data calculate the coordinates of the centre of mass of the foot and thigh given:

Ankle position = (84.9, 11.0) cm

Metatarsal position = (101.1, 1.3) cm

Greater trochanter = (72.1, 92.8) cm

Lateral femoral condyle = (86.4, 54.9) cm

Foot:

Centre of mass of foot is halfway between markers

$$\Rightarrow f = 0.5$$

If $\mathbf{r}_1 = (84.9, 11.0)$ & $\mathbf{r}_2 = (101.1, 1.3)$

Then

$$\begin{aligned} \mathbf{r} &= 0.5\mathbf{r}_1 + 0.5\mathbf{r}_2 \\ &= \frac{(84.9, 11.0) + (101.1, 1.3)}{2} \\ &= \frac{(186.0, 12.3)}{2} \\ &= (93.0, 6.15)cm \end{aligned}$$

Thigh:

Centre of mass is $0.433 \times$ length of thigh from greater trochanter.

If $\mathbf{r}_1 = (72.1, 92.8)$ & $\mathbf{r}_2 = (86.4, 54.9)$

$$\Rightarrow f = 0.433$$

$$\begin{aligned} \rightarrow \mathbf{r} &= (72.1, 92.8)(1 - 0.433) + (86.4, 54.9)(0.433) \\ &= (72.1, 92.8)(0.567) + (86.4, 54.9)(0.433) \\ &= (78.29, 76.39) \\ &= (78.3, 76.4)cm \end{aligned}$$

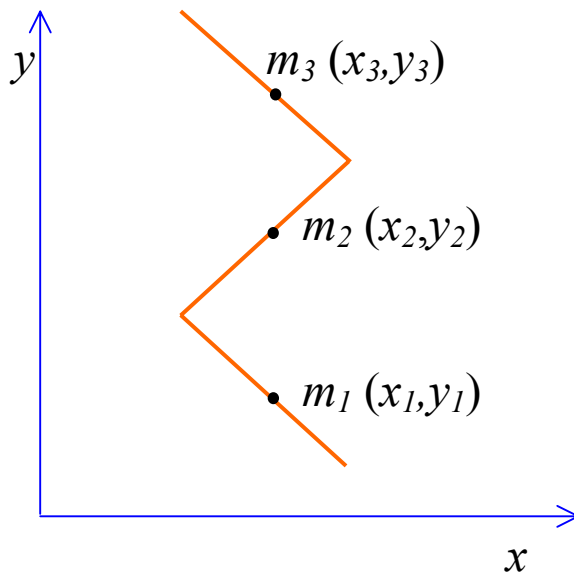
1.1.4 Centre of Mass of a Multisegment System.

With each body segment in motion the centre of mass of the total body is continuously changing with time.

⇒ It is necessary to recalculate it after each time interval.

This in turn requires a knowledge of the trajectories of the centre of mass of each body segment.

Consider a 3–segment system.



m_1 = mass of segment 1 with a centre of mass at (x_1, y_1)

m_2 = mass of segment 2 with a centre of mass at (x_2, y_2)

m_3 = mass of segment 3 with a centre of mass at (x_3, y_3)

$$M = m_1 + m_2 + m_3$$

Let the centre of mass of the system be at (x_0, y_0)

Then

$$x_0 = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M}$$

and

$$y_0 = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$

The centre of mass of the total body is a frequently calculated variable, however its usefulness in the assessment of human movement is quite limited.

Example:

The time history of the body centre of mass is often used to calculate energy changes of the body. But such calculation can be erroneous – because whole body centre of mass does not account for energy changes related to reciprocal movements of limbs, e.g., energy changes associated with the forward movement of one leg and the backward movement of the other will not be detected in a total body centre of mass calculation – because the centre of mass remains relatively unchanged.

– This will be discussed in the section on mechanical work, energy and power.

- Major use of body centre of mass is in sporting events such as jumping where the path of the centre of mass is critical to the success of the event because its trajectory is decided at takeoff.
- Also useful in studies of body posture and balance.

1.1.5 Mass, Moment of Inertia and Radius of Gyration.

Location of centre of mass of each segment is needed for an analysis of translational movement through space.

If accelerations are involved – we need to know the inertial resistance to such movements.

For linear motion:

$$F = ma$$

Where m = mass and is a measure of the segments ability to resist F .

For rotational motion:

$$\tau = I\alpha$$

Where τ = torque = moment of force

I = moment of inertia

α = angular acceleration

$\Rightarrow I$ is the constant of proportional that measures the ability of the segment to resist changes in angular velocity, i.e. to resist τ .

$$[\tau] = N.m, [I] = Kg.m^2, [\alpha] = rad.s^{-2}$$

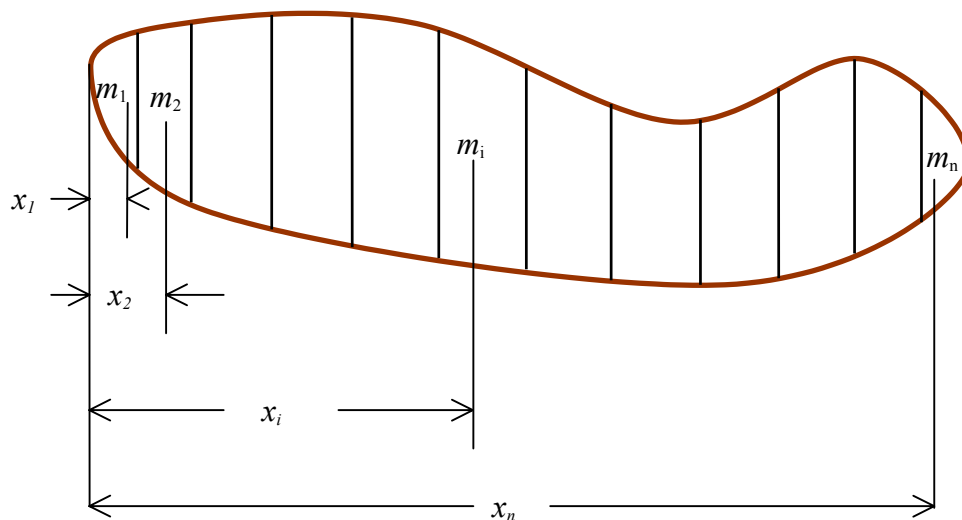
The value of I depends on the point about which the rotation occurs.

For a point mass m a distance r from an axis

$$I = mr^2 .$$

For a rigid body I is a minimum when rotation takes place about an axis through the centre of mass of the body.

Consider the following segment:



$$\begin{aligned} I &= m_1 x_1^2 + m_2 x_2^2 + \dots + m_n x_n^2 \\ &= \sum_{i=1}^n m_i x_i^2 \end{aligned}$$

\Rightarrow The further the mass is from the axis of rotation the greater its effect, e.g. fly-wheel.

The radius of gyration, k , is defined by:

$$k^2 = \frac{I}{M} \quad : \quad \boxed{\begin{aligned} I &= k^2 M \\ I &= \sum_i m_i x_i^2 \end{aligned}}$$

The radius of gyration is thus the distance from the axis at which a point mass M would have the same I as the body (or segment) of mass M .

$$x = \frac{1}{M} \sum_i m_i x_i = \text{position of centre of mass}$$

$$k = \sqrt{\frac{I}{M}} = \sqrt{\frac{1}{M} \sum_i m_i x_i^2} = \text{radius of gyration}$$

$$x_{\text{c of m}} = \frac{1}{M} \int x \, dm \quad \text{and} \quad I = \int x^2 \, dm$$

If I_0 is the moment of inertia of the segment about an axis through its centre of mass then:

$$I_0 = Mk_0^2$$

Where k_0 = radius of gyration about an axis through the centre of mass.

1.1.6 Parallel-Axis Theorem.

- Most body segments do not rotate about their centre of mass, but rather about the joint at either end.
- In vivo measures of I can only be taken about a joint centre.
- The relationship between I and I_0 is given by the parallel-axis theorem:

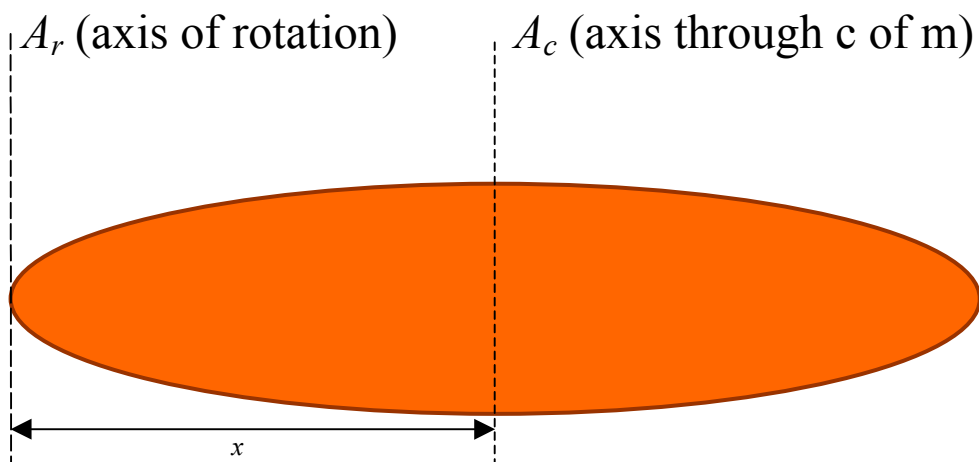
$$I = I_0 + Mx^2$$

Where,

I_0 = moment of inertia about the centre of mass

x = distance between center of mass and centre of rotation.

M = mass of segment



Note: A_r and A_c are parallel.

Example:

(a) A prosthetic leg has a mass of 3Kg and a centre of mass of 20cm from the knee joint. The radius of gyration (about the centre of mass) is 14.1cm.

Calculate I about the knee joint.

$$I_0 = Mk^2 = 3(0.141)^2 = 0.06 \text{ Kg.m}^2$$

$$I_k = I_0 + Mx^2 = 0.06 + 3(0.2)^2 = 0.18 \text{ Kg.m}^2$$

- (b) If the distance between the knee and hip joints is 42cm , calculate I_h for the prosthesis about the hip joint as the amputee swings through with a locked knee.

Now x = distance from centre of mass (of prosthetic leg) to hip joint

$$\text{So, } x = 20 + 42 = 62 \text{ cm}$$

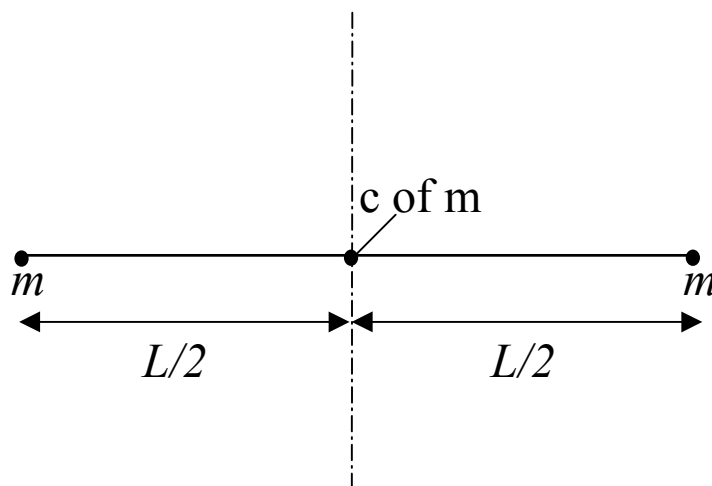
$$\therefore I_h = I_0 + Mx^2 = 0.06 + 3(0.62)^2 = 1.21 \text{ Kg.m}^2$$

Note: $I_h \sim 20I_0$.

Example:

Consider a rigid body consisting of two particles of mass m , connected by a weightless rod of length L .

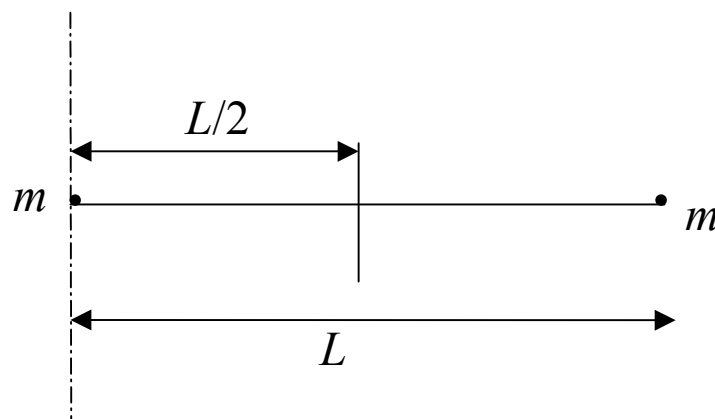
- (a) Axis through c of m



What is I about an axis perpendicular to rod and through the centre of mass of the rigid body?

$$\begin{aligned}
 I_0 &= \sum_i m_i x_i^2 \\
 &= m \left(\frac{1}{2} L \right)^2 + m \left(\frac{1}{2} L \right)^2 \\
 &= \frac{1}{2} m L^2
 \end{aligned}$$

(b) Axis through end of rod.



What is I of the body about an axis through one end of the rod and parallel to the first?

$$I = I_0 + Mx^2 \quad \text{where } M = m + m = 2m$$

$$\begin{aligned}
 I &= \frac{1}{2} m L^2 + (2m) \left(\frac{1}{2} L \right)^2 \\
 &= m L^2
 \end{aligned}$$

This result can be checked by direct calculation.

$$\begin{aligned}
 I &= \sum_i m_i x_i^2 \\
 &= m(0)^2 + m(L)^2 \\
 &= m L^2
 \end{aligned}$$

1.1.7 Use of Anthropometric tables and kinematic data.

Table 3.1 in Winter gives:

1. Segment mass as a fraction of body mass.
2. Centre of mass as a fraction of their lengths from either proximal or distal end.
3. Radius of gyration as a fraction of segment length about:
 - a) Centre of mass
 - b) Proximal end
 - c) Distal end
4. Density of segment

This information along with kinematic data can be used to calculate many variables needed for kinetic energy analysis.

1.1.7.1 Calculation of Segmental Masses and Centres of Mass

Example

Calculate the mass of the foot, shank, thigh and HAT and its location from the proximal or distal end assuming that the body mass of the subject is 80Kg.

Solution:

From table 3.1, mass fraction of various segments are:

$$\text{Foot} = 0.0145 \quad \Rightarrow \quad m_f = 0.0145 \times 80 = 1.16 \text{Kg}$$

$$\text{Shank} = 0.0465 \quad \Rightarrow \quad m_s = 0.0465 \times 80 = 3.72 \text{Kg}$$

$$\text{Thigh} = 0.10 \quad \Rightarrow \quad m_t = 0.10 \times 80 = 8.0 \text{Kg}$$

$$\text{HAT} = 0.678 \quad \Rightarrow \quad m_H = 0.678 \times 80 = 54.24 \text{Kg}$$

Direct measures of subject yield the following segment lengths:

$$\text{Foot} = 0.195 \text{m}$$

$$\text{Shank} = 0.435 \text{m}$$

$$\text{Thigh} = 0.410 \text{m}$$

$$\text{HAT} = 0.295 \text{m}$$

\Rightarrow

c of m of foot = $0.50 \times 0.195 = 0.098$ m between ankle
and metatarsal markers.

c of m of shank = $0.433 \times 0.435 = 0.188$ m below femoral
condyle marker.

c of m of thigh = $0.433 \times 0.410 = 0.178$ m below greater
trochanter marker.

c of m of HAT = $1.142 \times 0.295 = 0.337$ m above greater
trochanter marker.

1.1.7.2 Calculation of Total-body Centre of mass.

Consider a body composed of n segments, of mass

m_1, m_2, \dots, m_n .

Suppose the centre of mass of the i^{th} segment is (x_i, y_i, z_i)

then the total-body centre of mass in the x direction is:

$$\begin{aligned} x &= \frac{\sum_i m_i x_i}{\sum_i m_i} \\ &= \frac{1}{M} \sum_i m_i x_i \end{aligned} \quad (1)$$

If, $f_i = \frac{m_i}{M}$ = fraction of mass segment i to total body mass

Then,

$$x = \sum_i f_i x_i \quad (2)$$

Similarly

$$y = \sum_i f_i y_i$$

$$z = \sum_i f_i z_i$$

Equation (2) is easier to use than (1) because all we require is a knowledge of the fractions of total body mass and the coordinates of each segment's centre of mass.

Example.

A snap shot is taken of a person walking and an analysis of the position of the centre of mass of the body segments gives the following:

Segment	$x(m)$		$y(m)$	
	Right	Left	Right	Left
Foot	0.929	0.560	0.062	0.156
Leg	0.884	0.743	0.358	0.416
Thigh	0.863	0.846	0.773	0.760
½ Hat	0.791	0.791	1.275	1.265

Calculate the position of the centre of mass of the person.

$$f_{foot} = 0.0145$$

$$f_{leg} = 0.0465$$

$$f_{thigh} = 0.100$$

$$f_{1/2HAT} = 0.339$$

Fractional masses from
table 3.1 – column 3.

$$\begin{aligned}
 x &= \sum_{i=1}^8 f_i x_i = 0.0145(0.929 + 0.56) \\
 &\quad + 0.0465(0.884 + 0.743) \\
 &\quad + 0.10(0.863 + 0.846) \\
 &\quad + 0.339(0.791 + 0.791) \\
 &= 0.807m
 \end{aligned}$$

$$\begin{aligned}
 y &= \sum_{i=1}^8 f_i x_i = 0.0145(0.062 + 0.156) \\
 &\quad + 0.0465(0.358 + 0.416) \\
 &\quad + 0.10(0.773 + 0.760) \\
 &\quad + 0.339(1.275 + 1.265) \\
 &= 1.054m
 \end{aligned}$$

Hence the body centre of mass is at (0.807, 1.054) m.

It is not always possible to measure the centre of mass of every segment— especially if it is not in full view.

1.1.7.3 Calculation of Moment of Inertia.

Example

Calculate the moment of inertia of the leg (shank) about its centre of mass, its distal end and its proximal end for an 80Kg man with a leg of length 0.435m, given:

Radius of gyration/segment length is:

0.302 for c of m

0.528 for proximal end

0.643 for distal end

$$I = Mk^2$$

$$\begin{aligned}
 \text{mass of leg} &= \text{mass of body} \times f_{\text{leg}} \\
 &= 80 \times 0.0465 = 3.72Kg
 \end{aligned}$$

$$\begin{aligned}
 I_0 &= m_{leg} (k_0)^2 \\
 &= 3.72 \times (0.302 \times 0.435)^2 = 0.064 \text{ Kg.m}^2
 \end{aligned}$$

$$\begin{aligned}
 I_p &= m_{leg} (k_p)^2 \\
 &= 3.72(0.528 \times 0.435)^2 = 0.196 \text{ Kg.m}^2
 \end{aligned}$$

$$\begin{aligned}
 I_d &= m_{leg} (k_d)^2 \\
 &= 3.72(0.643 \times 0.435)^2 = 0.291 \text{ Kg.m}^2
 \end{aligned}$$

Note: I_p and I_d could be calculated using the parallel axis theorem:

$$I_p = I_o + m_{leg} x^2$$

x = distance between the centre of mass and proximal end.

$$= 0.433 \times 0.435 = 0.188 \text{ m}$$

∴

$$I_p = 0.064 + 3.72(0.188)^2 = 0.196 \text{ Kg.m}^2$$

$$I_p = I_o + m_{leg} x^2$$

x = distance between the centre of mass and distal end.

$$= 0.567 \times 0.435 = 0.247 \text{ m}$$

∴

$$I_p = 0.064 + 3.72(0.247)^2 = 0.291 \text{ Kg.m}^2$$

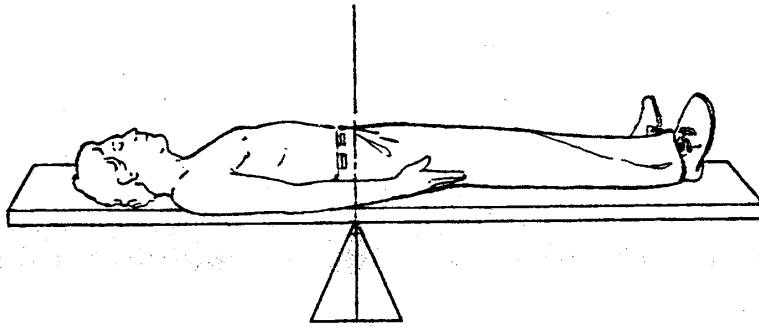
1.2 Direct experimental measures.

For more exact kinematic and kinetic calculations it may seem preferable to have directly measured anthropometric values.

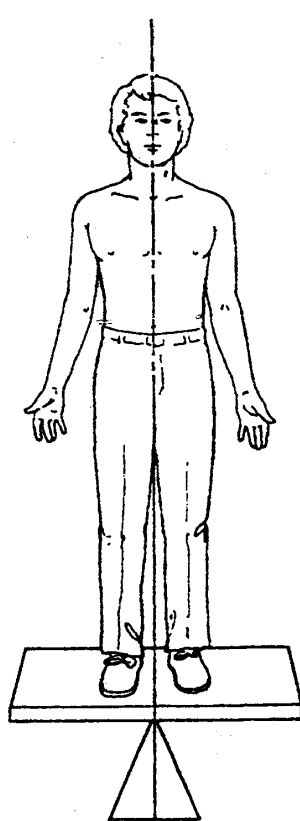
However the equipment and techniques that have been developed have limited capability and sometimes are not much of an improvement over the values obtained from tables.

1.2.1 Location of the Anatomical Centre of mass of the body.

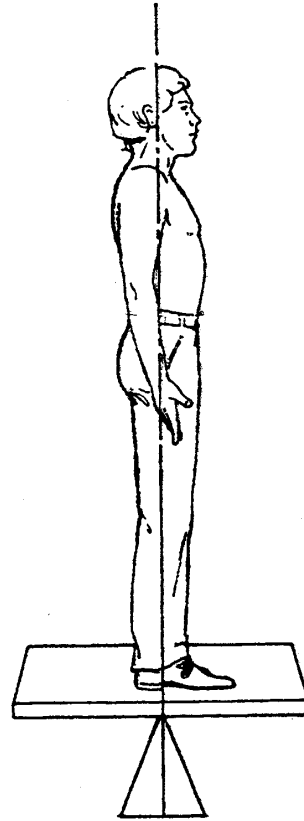
How can we locate the anatomical centre of mass of the body?



Top and bottom portions balance



Right and left portions balance

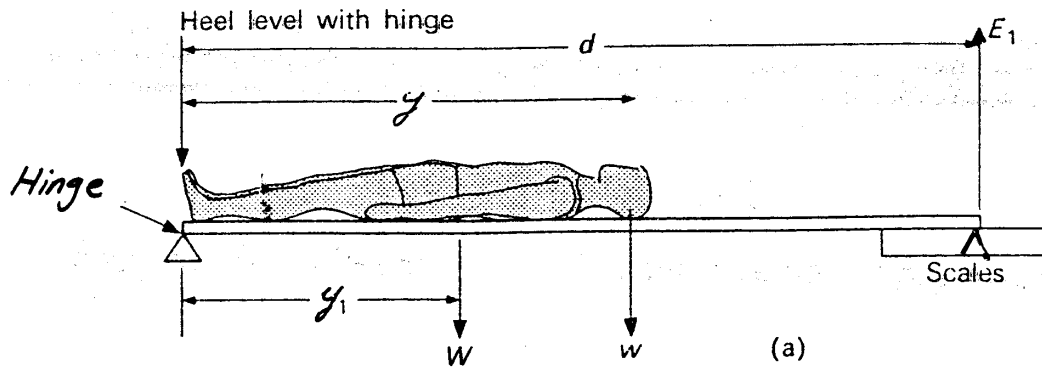


Front and back portions balance

By balancing the body!

The anatomical centre of mass of the body can be located by using a balance (or reaction) board and a set of scales.

Consider the following:



$$\sum \tau = 0$$

Consider rotation about “hinge”

d = length of board – actually length between supports.

y = distance from hinge to centre of mass of board.

y_1 = distance from hinge to centre of mass of body.

E_1 = scale reading = reaction force at scale support.

W = weight of body.

w = weight of board.

Taking torques about hinge \Rightarrow

$$Wy_1 + wy = E_1d$$

$$\Rightarrow y_1 = \frac{E_1d - wy}{W}$$

Do we need to measure w and y ?

Without the person on the board $\sum \tau \Rightarrow$

$$wy = E_0d$$

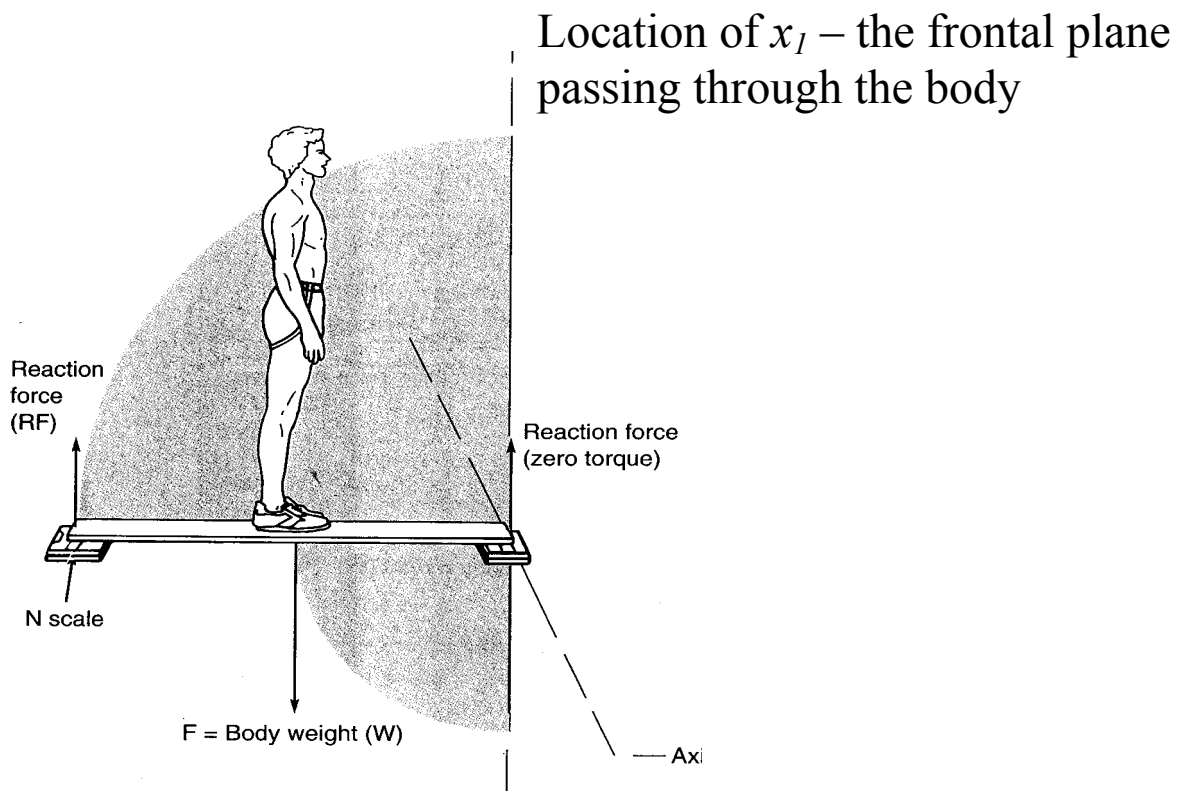
Where E_0 = scale reading with board only.

$$\therefore y_1 = \frac{(E_1 - E_0)d}{W}$$

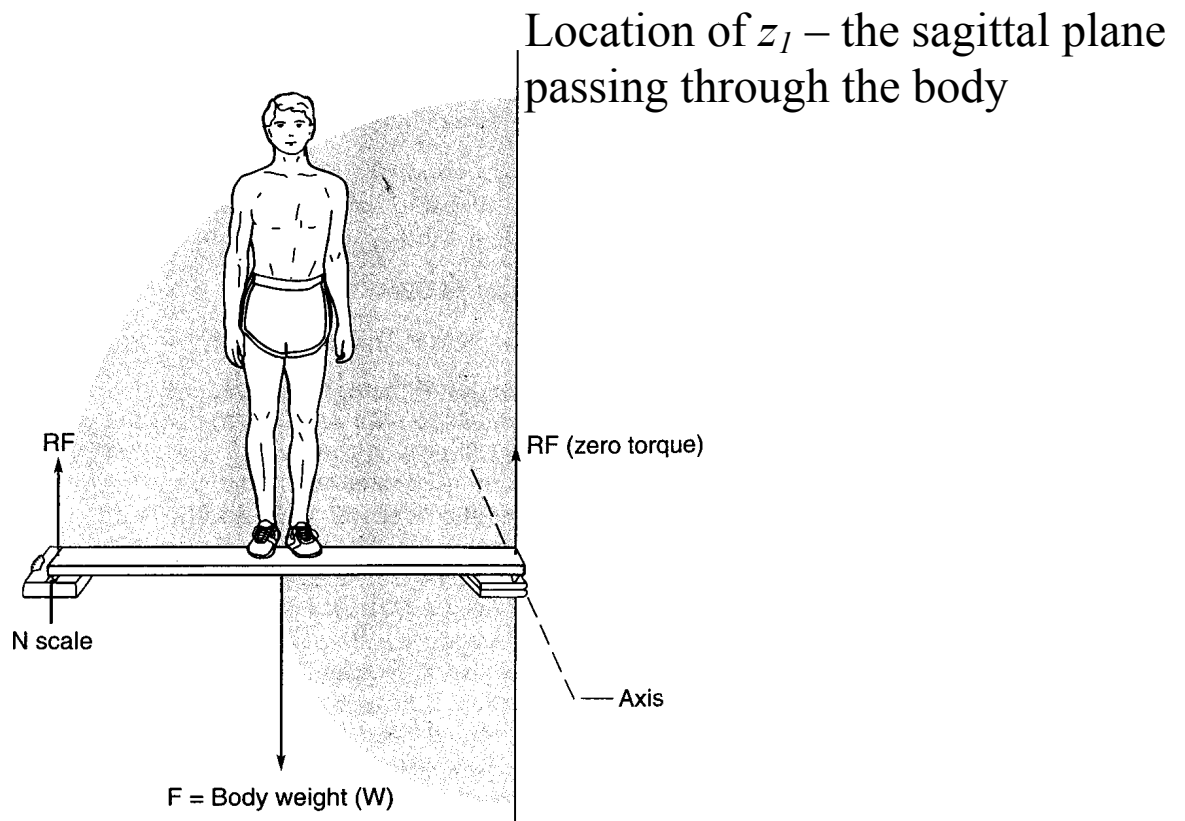
Note: If heel does not coincide with position of hinge then the difference will have to be allowed for.

The above locates the transverse plane passing through centre of mass.

The frontal plane passing through centre of mass (x_1) can be located by standing on board and facing along the axis of the board.



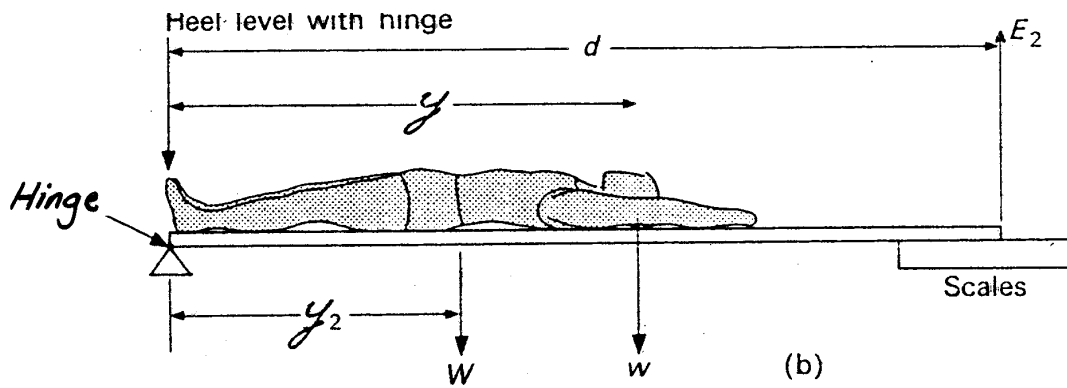
The sagittal plane passing through centre of mass (at z_1) can be located in a similar fashion by standing on board and facing perpendicular to axis of board.



What happens when the subject moves a body segment?

The position of the body centre of mass changes!

Suppose the subject moves both arms over head.



Let body centre of mass move to y_2 and the new scale reading be E_2 .

$$y_2 = \frac{(E_2 - E_0)d}{W}$$

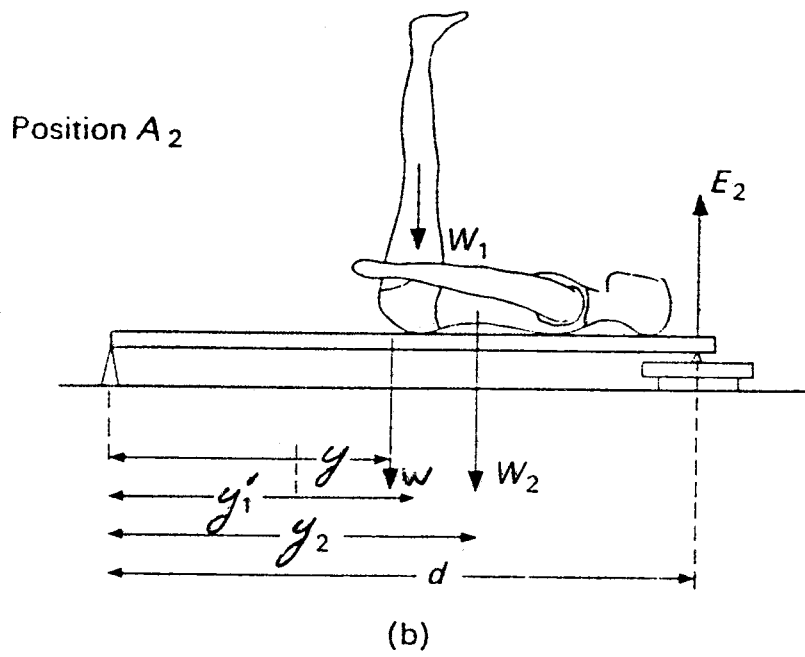
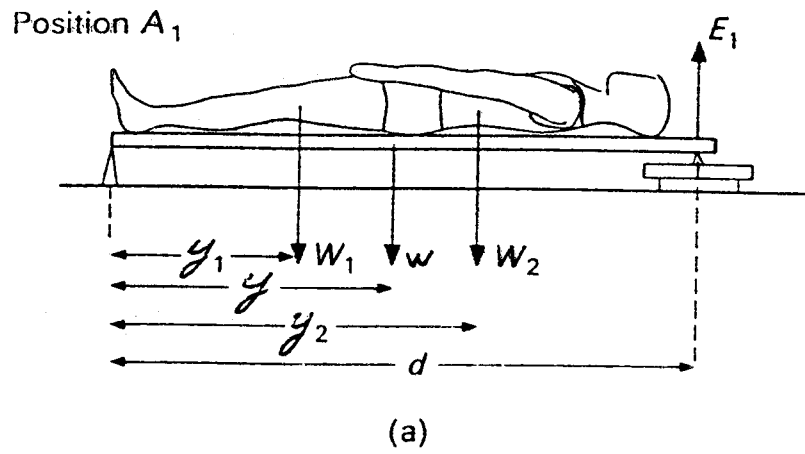
$$\rightarrow \Delta y = y_2 - y_1 = \frac{(E_2 - E_1)d}{W}$$

which represents the vertical shift in the centre of mass.

($\sim 7cm$).

1.2.2 Determination of Segmental Masses.

Segment weights can be obtained by the following procedure, where we consider the determination of the weight of the two thighs and lower legs.



Body segment weights.

Here

W_1 = weight of both thighs and lower legs

W_2 = weight of rest of body

y_1 = distance from hinge (fulcrum) to centre of mass of thighs and lower legs, in position A_1 .

y_1' = distance from hinge to centre of mass of thighs and lower legs, in position A_2 .

y_2 = distance from hinge to centre of mass of rest of body.

y = distance from hinge to centre of mass of board

d = distance from hinge fulcrum to scale's fulcrum.

E_1 = scale's reading in position A_1 .

E_2 = scales reading in position A_2 .

In position A_1 , $\sum \tau = 0$ about hinge \Rightarrow

$$W_1 y_1 + W_2 y_2 + wy = E_1 d \quad (1)$$

In position A_2 , $\sum \tau = 0$ about hinge \Rightarrow

$$W_1 y_1' + W_2 y_2 + wy = E_2 d \quad (2)$$

Eqn(2) – Eqn(1) \Rightarrow

$$W_1 y_1' - W_1 y_1 = E_2 d - E_1 d$$

\Rightarrow

$$W_1 = \frac{(E_2 - E_1) d}{y_1' - y_1}$$

Now E_1 , E_2 and d are readily measured.

What about y_1 and y_1' ?

$y_1' - y_1 =$ distance the centre of mass of legs has moved

$=$ distance between centre of mass of legs and

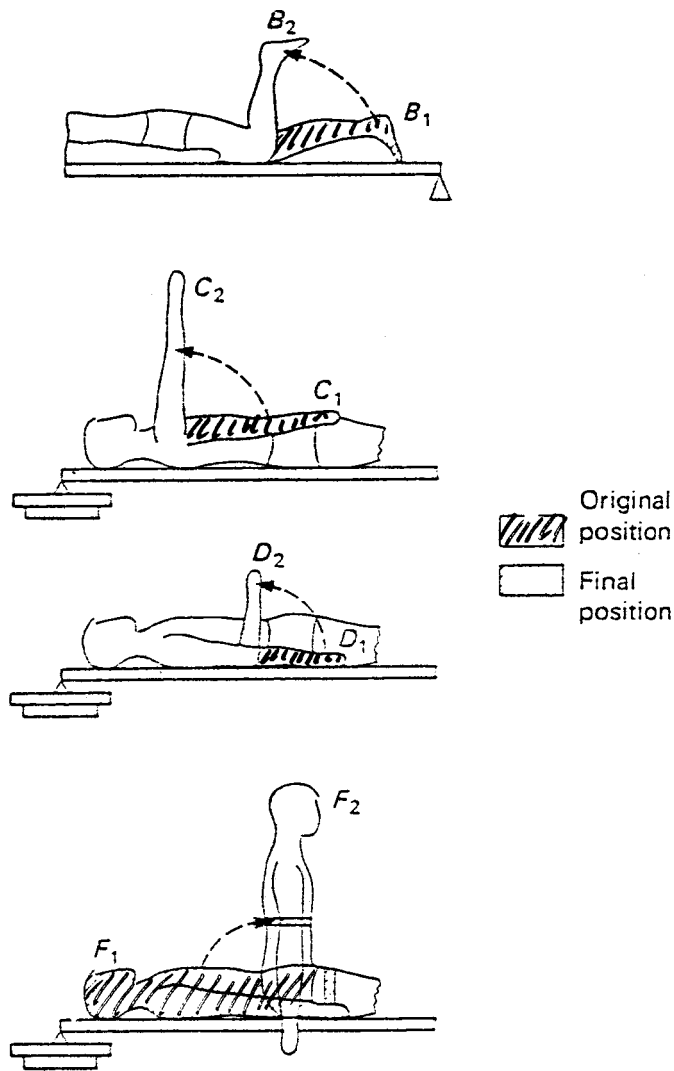
hip joint.

– Get from anthropometric tables

– Major source of error in the above calculations.

The same procedure can be used to measure:

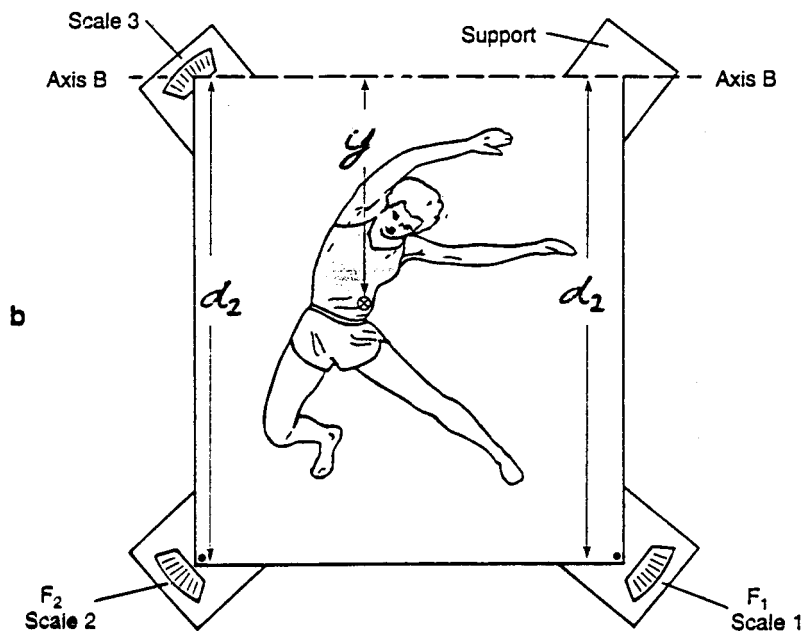
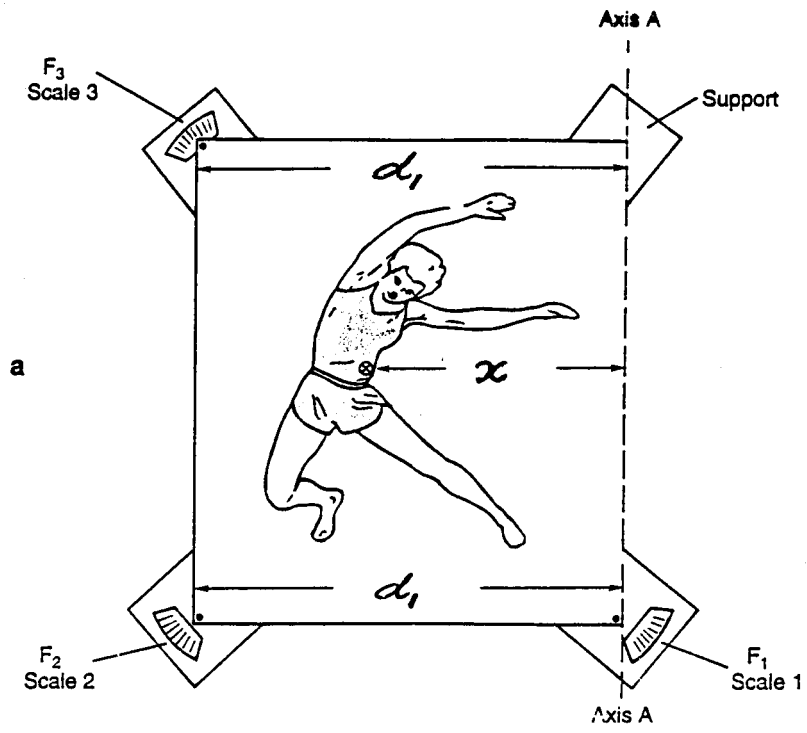
- Weight of lower leg and foot
- Weight of whole arm
- Weight of forearm and hand
- Weight of HAT



Positions $B_1 - B_2 : C_1 - C_2 : D_1 - D_2$ etc. which yield the segment weights of the lower leg and foot, the arm and hand, the forearm and hand and the head, trunk and arm.

1.2.3 Determination of centre of mass in two direction simultaneously.

By using three scales, a support and a large square or rectangular body, the centre of mass in two directions can be determined simultaneously.



Procedure:

- Place board in scales and support (board is horizontal)
- Zero scales

Apply $\sum \tau = 0$ about axis A and axis B .

About axis A :

$$F_3 d_1 + F_2 d_1 = Wx$$

$$\rightarrow x = \frac{(F_3 + F_1) d_2}{W}$$

About axis B :

$$F_2 d_2 + F_1 d_2 = Wy$$

$$\rightarrow y = \frac{(F_2 + F_1) d_2}{W}$$

Where: F_1 = reading on scale 1

F_2 = reading on scale 2

F_3 = reading on scale 3

x = perpendicular distance from axis A to centre of mass.

y = perpendicular distance from axis B to centre of mass.

and d_1 and d_2 are length and width of board.

The above procedure has been used (is used) to determine centre of mass of athletes or various activities.

Procedure:

Selected frames of a film are projected on the “board” to produce life size images. The outline of the image is traced over. The board is then placed on the scales and support and the subject gets into the marked position. Measurement of the centre of mass is then completed.
→ Difficulty in reproducing the exact performance position. See examples below

Eg 1

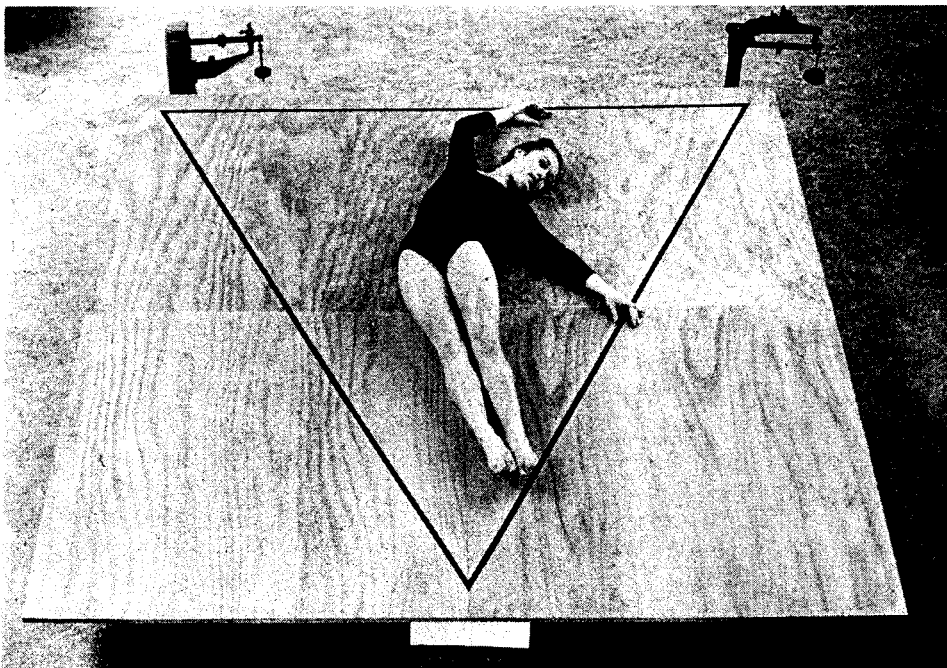


Figure 129. Large reaction board used to determine the location of the center of gravity in two dimensions.

Eg 2

THE CENTRE OF GRAVITY OF THE HUMAN BODY

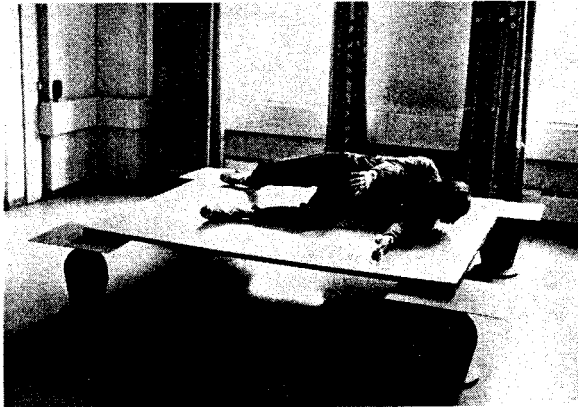


FIG. 3.11 *Full centre of gravity board.*

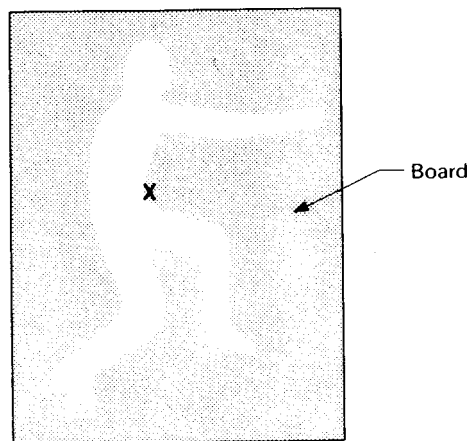
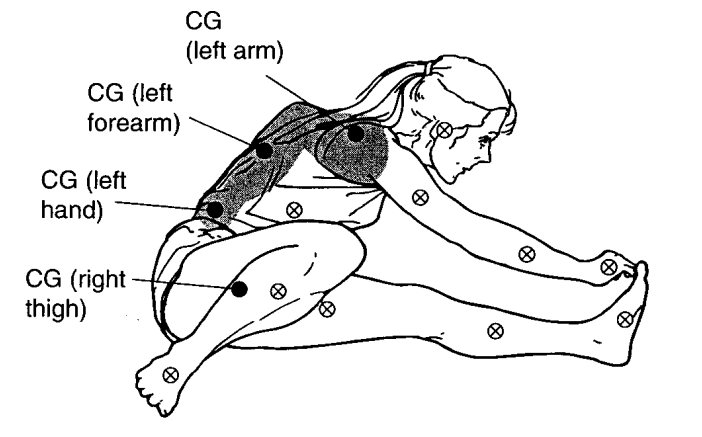


FIG. 3.12 *Position of the centre of gravity for subject in position shown in fig. 3.11.*

Today researches tend to favor the “segmental method” discussed earlier (see calculation of total-body centre of mass). The draw back is that it relies on anthropometric tables (for the fractional masses f_i – see example below).

$$x = \sum_{i=1}^n f_i x_i \quad y = \sum_{i=1}^n f_i y_i \quad (2D \text{ component form})$$

$$\mathbf{r} = \sum_{i=1}^n f_i \mathbf{r}_i \quad (\text{vector form})$$



A hurdler with segmental CGs identified.

Segmental method of determining total-body centre of mass (CG = centre of gravity \equiv centre of mass)

1.2.4 Experimental determination of I for distal segment.

Given, $\tau = I\alpha$ can you think of a way to determine I for a distal body segment?

1.3 Muscle anthropometry.

In order to calculate forces produced by individual muscles during movement, we need to dimensions of the muscles themselves.

If muscles of same group share a load – they probably do so proportionally to their relative X-sectional areas.

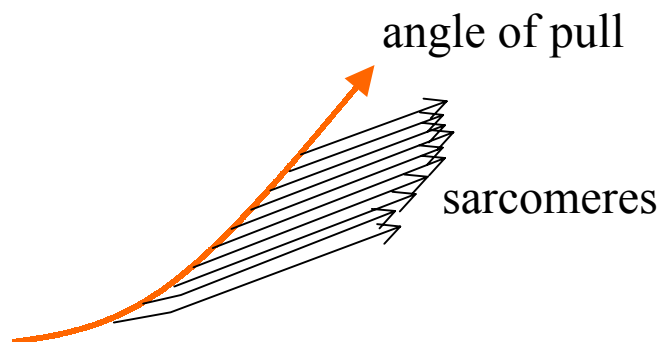
$$\rightarrow \text{Force generation} \propto \text{X-area}$$

In addition the mechanical advantage of each muscle can be different depending upon:

1. Moment arm length at its origin and insertion.
2. On other structures beneath the muscle or tendon which alter the angle of pull (of tendon) e.g. patella

1.3.1 Cross-Sectional Area of Muscle.

The functional or physiologic X-area (PCA) of a muscle is a measure of the number of sarcomeres in parallel with the angle of pull of the muscle.



Pennate muscles - fibers act at an angle to long axis.

⇒ not as effective as fibers in a parallel-fibered muscle.

In parallel-fibered muscle:

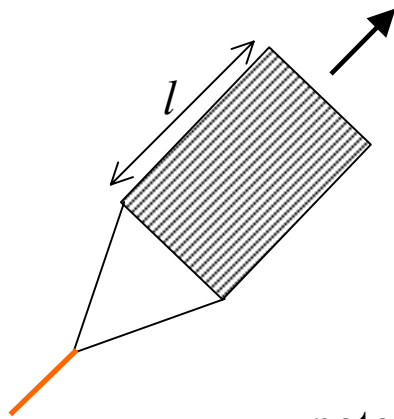
$$PCA = \frac{m}{\rho l} \text{ (cm}^2\text{)}$$

where:

m = mass of muscle fibers (*grams*)

ρ = density of muscle fibers $\sim 1.056 \text{ g/cm}^3$

l = length of muscle fibers (*cm*)



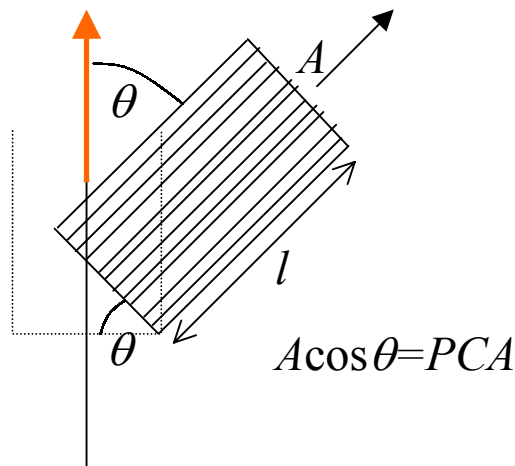
$$\begin{aligned} m &= \text{volume} \times \text{density} \\ &= l \times A \times \rho \\ \rightarrow A &= PCA = \frac{m}{\rho l} \end{aligned}$$

note: PCA perpendicular to line of pull.

In pennate muscle:

$$PCA = \frac{m \cos \theta}{\rho l} \text{ (cm}^2\text{)}$$

θ = pennation angle (increases as muscle shortens)



$$V = A \times l$$

$$m = A \times l \times \rho$$

$$A = \frac{m}{l\rho}$$

but now $PCA = A \cos \theta$

$$\Rightarrow A = \frac{PCA}{\cos \theta}$$

$$\therefore PCA = \frac{m \cos \theta}{\rho l}$$

Example:

Use the data on leg muscle architecture to determine whether knee flexor or knee extensors produce the greater force.

Knee flexors:

Biceps femoris (short and long)	PCA = 12.8 cm ²
Semimembranosus	PCA = 16.9 cm ²
Semitendinosus	PCA = 5.4 cm ²
Gastrocnemius	PCA = 32.4 cm ²
Total	PCA = 67.5 cm ²

Knee extensors:

Rectus femoris	PCA = 12.7 cm ²
Vastus intermedius	PCA = 22.3 cm ²
Vastus lateralis	PCA = 30.6 cm ²
Vastus medialis	PCA = 21.1 cm ²

Total

$$PCA = 86.7 \text{ cm}^2$$

⇒ Knee extensors will generate greater force

Does this mean the knee extensors are stronger?

Maybe – maybe not. The torques generated by each group will decide that.

Note in table 3.4 of Winter PCA is given as a % of total X-section area for muscles crossing a joint ⇒ multi-joint muscles may have different % at different joints.

1.3.2 Changes in Muscle Length during Movement.

A few studies have investigated the changes in muscle lengths as a function of joint angles.

Example:

Grieve et al (1978) studied eight cadavers for changes in length of gastrocnemius as a function of knee and ankle angle.

Assuming:

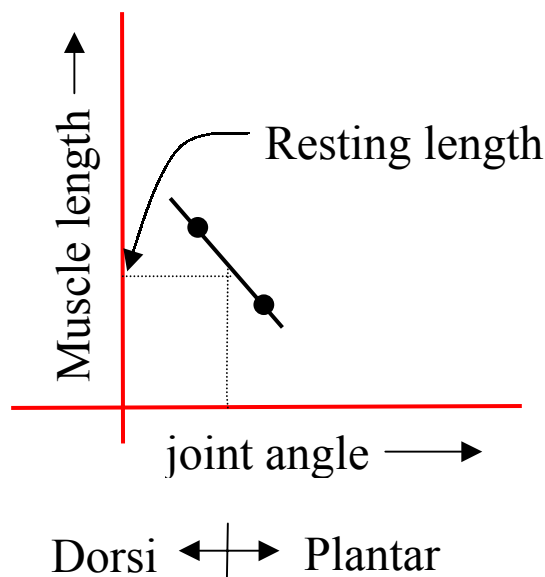
Gastrocnemius resting length = knee flexed at 90° and
ankle in intermediate position.

They found (for ankle):

40° plantar-flexion → muscle shortened 8.5%

20° dorsi-flexion → muscle lengthened 4%

with a linear response between.



For knee:

At full extension → gastroc lengthened by 6.5%

150° flexion → gastroc shortened by 3%

1.3.3 Muscle stress (Force per unit X–area)

A wide range of stress values have been reported for skeletal muscles – mainly under isometric contraction.

Values range from 20 – 100 N/cm^2 .

Higher values tend to be recorded for pennate muscles.

For quadriceps both dynamic and isometric stresses have been determined:

Dynamic stresses $\sim 70N/cm^2$ have been reported during running and jumping (based on peak torque production).

Isometric stresses $\sim 100N/cm^2$ reported.

1.3.4 Multi-joint Muscles.

- A large number of muscles in the body cross more than one joint, e.g.
 - hamstrings
 - rectus femoris
 - gastrocnemius
- Fiber length of many of these muscles may be insufficient to allow complete range of movement over both joints, simultaneously. $\Rightarrow ?$

- Consider the action of the rectus femoris (RF) during early swing in running. It shortens as a result of hip flexion (thigh swings forward) and lengthens as a result of knee flexion (leg swings back).

Tension in the RF simultaneously creates a hip flexor torque (positive work) to accelerate the thigh and a knee extensor torque (negative work) to decelerate the swinging leg and start acceleration forward.

\Rightarrow The (absolute) change in muscle length is reduced compared with two equivalent single-joint muscles. Excessive positive and negative work within the muscle can be reduced.

The two-joint muscle could be totally isometric in the above e.g. \rightarrow transfer of energy from leg to pelvis.

Exercise

- 1– Explain contraction of gastrocnemius during push-off.
- 2– Explain contraction of biceps during a punch.

Section 2 Kinetics: Forces and Torques

2.0 Biomechanical models.

- Kinematics studies movements without regard to the forces that cause it.
- Kinetics studies these forces and the resulting energetics.
- Transducers can be implanted surgically to measure the force exerted by a muscle at the tendon – animal experiments only!

For humans we attempt to calculate these forces indirectly using the available kinetic, kinematic and anthropometric data – the process is called link-segment modeling.

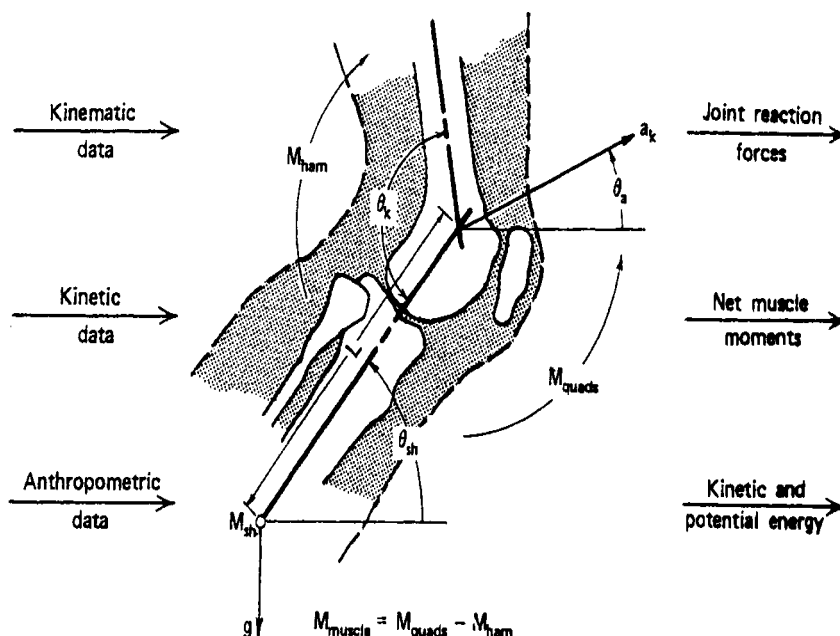


Figure 4.1 Schematic diagram of the relationship between kinematic, kinetic, and anthropometric data and the calculated forces, moments, energies, and powers using an inverse solution of a link-segment model.

Given:

Kinematic description and accurate anthropometric measures and external forces → joint reaction forces and muscle torques.

This is called an **inverse solution** – a very powerful tool in gaining insight into the net summation of all muscle activity at each joint.

Such information is useful for coaches, surgeons and therapists in diagnostic assessment – the effect of training, therapy, or surgery is evident at this level of assessment, but is often obscured in the original kinematics.

2.0.1 Link-Segment Model Development.

- Validity of any assessment is only as good as the model itself.
- Accurate measures of segment masses, centres of mass, joint centres and moments of inertia are required. Such data can be obtained from statistical tables or measured directly (see section on anthropometry).

- The model makes the following assumptions:
 1. Each segment has a fixed mass located as a point mass at its centre of mass.
 2. The location of each segment's centre of mass remains fixed during the movement.
 3. The joints are considered to be hinge or ball and socket.
 4. I of each segment about its centre of mass (or proximal or distal joints) is constant during the movement.
 5. The length of each segment remains constant during the movement.

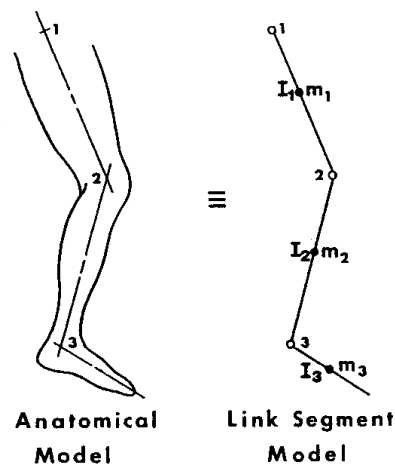


Figure 4.2 Relationship between anatomical and link-segment models. Joints are replaced by hinge (pin) joints and segments are replaced by masses and moments of inertia located at each segment's center of mass.

The above figure shows the relationship between anatomical and link-segment models.

Note:

- Segment masses m_1 , m_2 and m_3 are considered to be concentrated at points,
- I_1 , I_2 and I_3 are fixed.
- L_1 , L_2 and L_3 are fixed.

2.0.2 Forces Acting on the Link-Segment Model.Gravitational forces.

g acts vertically down through the centre of mass of each segment with force = mg

Ground reaction or external forces.

External forces are measured by force transducers. Such forces are distributed over an area of the body (e.g. ground reaction forces on feet) and apply a 'pressure' to the body part. In order to represent such a 'pressure' by a single vector, they must be considered to act at a point – the centre of pressure.

Centre of pressure can be calculated from force plates.

Muscle and ligament forces.

The net effect of muscle activity at a joint is calculated in terms of net muscle torque.

- When co-contractions take place the analysis yields only the net effect of agonistic and antagonistic muscles.
- In addition any friction effects at the joints or within the muscle cannot be separated from this net value. Increased friction merely reduces the effective ‘muscle’ torque.
- At the extreme range of movement (of any joint) passive structures such as ligaments come into play to contain the range. Torques produced by these tissues will add to or subtract from those generated by the muscles.
- The contribution of passive structures can only be determined if the muscle is silent.

2.0.3 Joint reaction forces and bone-on-bone forces.

The three forces above constitute all the forces acting on the total body itself.

However our analysis examines the segments one at a time.

Therefore we must calculate the reaction between segments.

A free-body diagram is required for each segment.

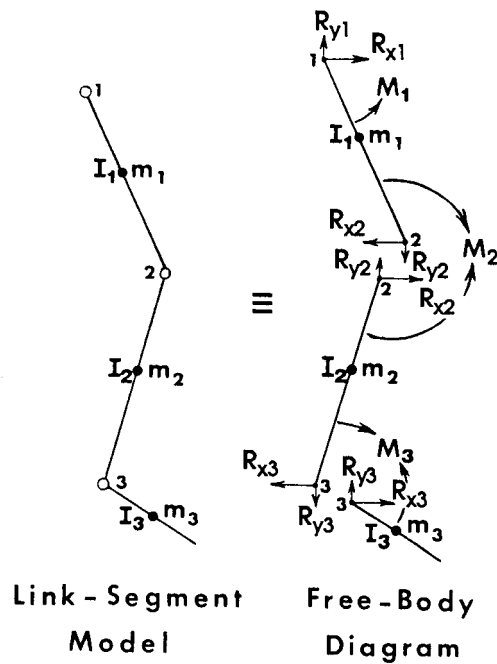


Figure 4.3 Relationship between the free-body diagram and the link-segment model. Each segment is “broken” at the joints, and the reaction forces and moments of force acting at each joint are indicated.

The original link-segment model is broken into its segmental parts. The breaks are made at the joints and the forces that act across each joint must be shown on the resultant free-body diagram. We can now look at each segment and calculate the unknown joint reaction forces.

Newton’s third law: for each action there is an equal and opposite reaction.

“If one object exerts a force \mathbf{F} on a second, then the second object exerts an equal but opposite force $-\mathbf{F}$ on the first”

\Rightarrow there are equal and opposite forces acting at each joint

Consider joint 2 in the above diagram:

Let $\mathbf{R}_{2,2}$ = reaction force at joint 2 on segment 2

and $\mathbf{R}_{2,1}$ = reaction force at joint 2 on segment 1

Newton's third law $\Rightarrow \mathbf{R}_{2,1} = -\mathbf{R}_{2,2}$

\therefore If

$$\mathbf{R}_{2,2} = (R_{2,2x}, R_{2,2y})$$

and

$$\mathbf{R}_{2,1} = (R_{2,1x}, R_{2,1y})$$

then

$$\mathbf{R}_{2,1} = (-R_{2,2x}, -R_{2,2y})$$

Since we need only refer to the joint, the above notation can be simplified.

Setting $R_{2,2x} = R_{2x}$ and $R_{2,2y} = R_{2y}$, \Rightarrow

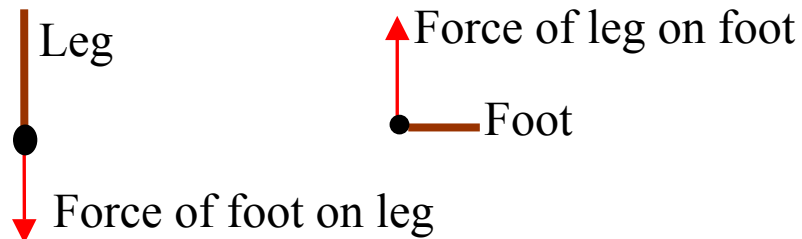
$$\mathbf{R}_{2,2} = (R_{2x}, R_{2y}) \text{ and } \mathbf{R}_{2,1} = -(R_{2x}, R_{2y}) = (-R_{2x}, -R_{2y})$$

Note: In the free-body diagram above, the opposite directions of the reaction force components on the two segments indicates that the forces are equal & opposite across the joint.

Example:

When leg is held off the ground in a static condition, weight of foot exerts a downward force on tendons and ligaments crossing the ankle joint. This is seen as a downward force acting on the leg, equal to the weight of the foot.

The reaction force at the ankle joint is the upward force exerted by the leg on the foot through the same connective tissue. This force is also equal in magnitude to the weight of the foot.



Don't confuse joint reaction forces with joint bone-on-bone forces!

Bone-on-bone forces are forces acting across the articulating surfaces and include the effect of muscle activity.

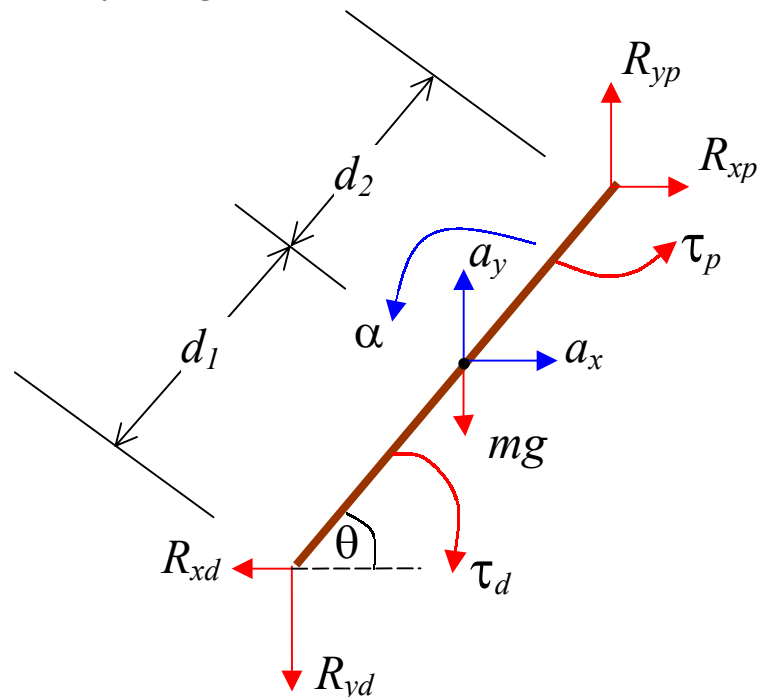
Actively contracting muscles pull the articulating surfaces together, creating compressive forces and sometimes shear forces (they may pull the joint apart – dislocating component) – See page 79 of Winter for more details.

2.1 Basic Link Segment Equations – The free-body diagram.

- Each body segment acts independently under the influence of reaction forces and muscle torques, which act at either end + the forces due to gravity.

Consider the planar movement of a segment in which the kinematics, anthropometrics and reaction forces at the distal end are known.

The free-body diagram is.



Known:

(a_x, a_y) = acceleration of segment centre of mass.

θ = angle of segment in plane of movement.

α = angular acceleration of segment in plane of movement.

(R_{xd}, R_{yd}) = reaction forces acting at distal end of segment
 (usually determined from a prior analysis of
 the proximal forces acting on distal segment).

τ_d = net muscle torque acting at distal joint (usually
 determined from an analysis of the proximal muscle
 acting on distal segment).

Unknown:

(R_{xp}, R_{yp}) = reaction forces acting at proximal joint.

τ_p = net muscle torque acting at proximal joint.

Equations:

Newton's second law of linear motion \Rightarrow

$$\sum \mathbf{F} = m\mathbf{a}$$

For 2-dimensional motion this can be written as

$$\sum F_x = ma_x \quad (1)$$

and

$$\sum F_y = ma_y \quad (2)$$

From the free-body diagram equations (1) & (2) lead to

$$R_{xp} - R_{xd} = ma_x \quad \text{for motion in the x direction}$$

$$R_{yp} - R_{yd} - mg = ma_y \quad \text{for motion in the y direction}$$

Where:

- For the x motion; forces to the right go into equation (1) as +ve, while for the forces to the left go into equation (1) as -ve.
- For the y motion; forces up go into equation (2) as +ve, while forces down go into equation (2) as -ve.

Expressing the unknowns on the right-hand side (LHS) and the knowns on the left-hand side (RHS) we have

$$R_{xp} = R_{xd} + ma_x$$

$$R_{yp} = R_{yd} + m(g + a_y)$$

Thus from Newton's 2nd law we can find the reaction force at the proximal end of the segment.

To find the muscle torque at the proximal end we use

Newton's 2nd law of rotational motion, i.e., $\sum \tau = I\alpha$

Taking rotation about centre of mass, this is written as;

$$\sum \tau = I_0\alpha \quad (3)$$

From the free-body diagram we sum over all torques to obtain

$$R_{yp}d_2 \cos\theta - R_{xp}d_2 \sin\theta + \tau_p + R_{yd}d_1 \cos\theta - R_{xd}d_1 \sin\theta - \tau_d = I_0\alpha$$

\Rightarrow

$$(R_{yp}d_2 + R_{yd}d_1)\cos\theta - (R_{xp}d_2 + R_{xd}d_1)\sin\theta + \tau_p - \tau_d = I_0\alpha$$

Where:

- Anti-clockwise torques; go into equation (3) as +ve.
- Clockwise torques; go into equation (3) as -ve.

Note that we can not calculate τ_p until R_{xp} and R_{yp} have been calculated. That is, we must solve equations (1) & (2) before equation (3).

Putting the unknown on the LHS and collecting the unknowns on the RHS, the torque equation becomes,

$$\tau_p = I_0\alpha - (R_{yp}d_2 + R_{yd}d_1)\cos\theta + (R_{xp}d_2 + R_{xd}d_1)\sin\theta + \tau_d.$$

Example 1:

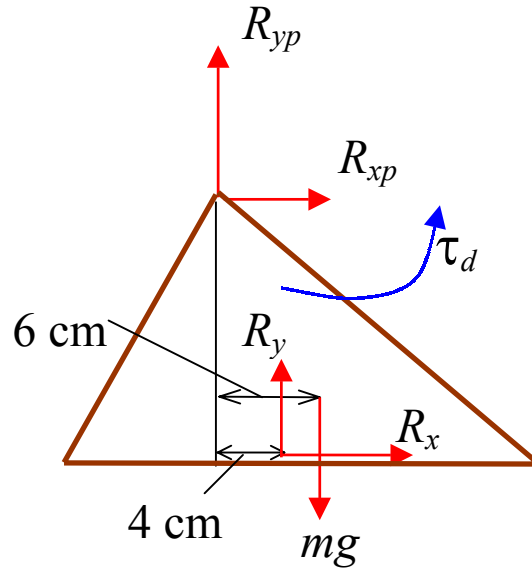
In a static situation, a person stands on one foot on a force plate. The ground reaction force acts 4cm anterior to the ankle joint.

The subject mass is 60Kg

The mass of the foot is 0.9Kg with a centre of mass 6cm anterior to the ankle.

Calculate the joint reaction forces and net muscle torque at the ankle.

Free-body diagram



Note: by convention R_y is up and R_x is to the right.

$$R_x = 0 \text{ (person is not moving)}$$

$$R_y = 60 \times 9.8 = 588 \text{ N}$$

$$1 - \sum F_x = ma_x$$

$$\Rightarrow R_{xp} + R_x = 0 \Rightarrow R_{xp} + 0 = 0 \Rightarrow R_{xp} = 0$$

$$2 - \sum F_y = ma_y$$

$$\Rightarrow R_{yp} + R_y - mg = ma_y$$

$$\Rightarrow R_{yp} + 588 - 0.9 \times 9.8 = 0$$

$$\Rightarrow R_{yp} = -588 + 8.82 = -579.8 \text{ N}$$

Negative sign \Rightarrow force is acting down.

3– Torque about centre of mass

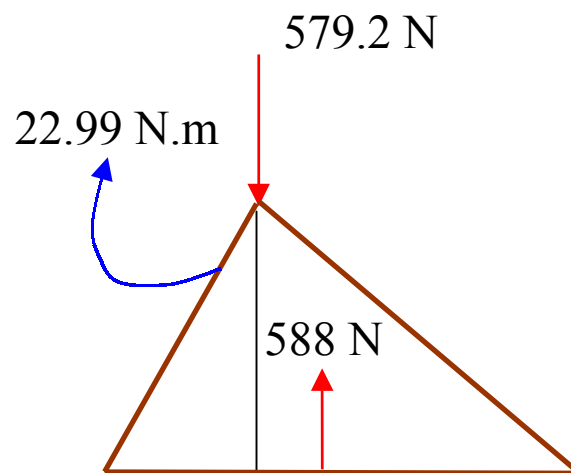
$$\sum \tau = I_0 \alpha$$

$$\Rightarrow \tau_p - R_y \times 0.02 - R_{yp} \times 0.06 = I_0 \alpha = 0$$

$$\begin{aligned} \Rightarrow \tau_p &= R_y \times 0.02 + R_{yp} \times 0.06 \\ &= (588 \times 0.02) + (-579.2 \times 0.06) \end{aligned}$$

$$\Rightarrow \tau_p = -22.99 \text{ N.m}$$

The actual situation is;



\Rightarrow

- A muscle torque of 22.99 N.m acts on the ankle
- Plantar-flexors are active at the ankle joint to maintain the static position.

Plantar flexors – Gastrocnemius, Soleus, Tibialis posterior, Peroneus longus, Peroneus brevis, Plantaris, Flexor digitorum longus, Flexor hallucis longus.

Example 2:

From the data collected during the swing of the foot, calculate the muscle torque and reaction force at the ankle.

We have:

Subject mass = 80Kg

Ankle-metatarsal length = 20.0cm

Video analysis gives the following info:

Angle (θ) of axis of foot with horizontal = -80°

(i.e., 80° clockwise from +ve x-axis.)

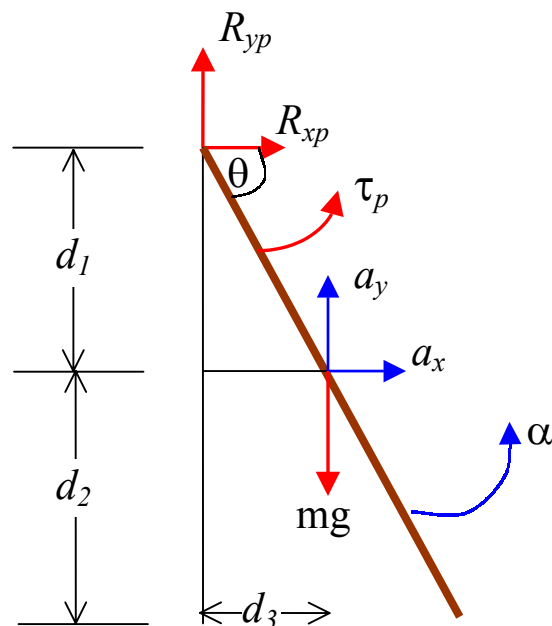
$$a_x = 9.07 \text{ m/s}^2$$

$$a_y = -6.62 \text{ m/s}^2$$

$$\alpha = 21.69 \text{ rad/s}^2$$

Solution:

Free-body diagram



Solution

Anthropometry:

$$M = 80 \text{ Kg}$$

$$\text{Foot length} = 20.0 \text{ cm}$$

Using anthropometric tables \rightarrow

$$m = 0.0145 \times 80 = 1.16 \text{ Kg}$$

$$k_0 = 0.475 \times 0.20 = 0.095 \text{ m} \Rightarrow$$

$$I_0 = 1.16(0.095)^2 = 0.0105 \text{ Kg.m}^2$$

$$\text{distance ankle joint to centre of mass} = 0.5 \times 20 = 10 \text{ cm}$$

 \Rightarrow

$$d_1 = 10 \sin 80 = 9.85 \text{ cm} (= d_2)$$

$$d_3 = 10 \cos 80 = 1.74 \text{ cm}$$

$$1 - \sum F_x = ma_x$$

$$\begin{aligned} \Rightarrow R_{xp} &= 1.16 \text{ Kg} \times 9.07 \text{ m/s}^2 \\ &= 10.52 \text{ N} \end{aligned}$$

$$2 - \sum F_y = ma_y$$

$$\Rightarrow R_{yp} - mg = ma_y$$

$$\begin{aligned} \Rightarrow R_{yp} &= m(g - a_y) \\ &= 1.16(9.8 - 6.62) = 3.69 \text{ N} \end{aligned}$$

3 – At centre of mass of foot $\sum \tau = I_0 \alpha$

$$\Rightarrow \tau_p - R_{xp} \times d_1 - R_{yp} \times d_3 = I_0 \alpha$$

$$\Rightarrow \tau_p = I_0 \alpha + R_{xp} d_1 + R_{yp} d_3$$

$$= 0.0105 \times 21.69 + 10.52 \times 0.0985 + 3.69 \times 0.0174$$

$$= 0.23 + 1.04 + 0.06$$

$$= 1.33 \text{ N.m}$$

Note:

1. The horizontal reaction force of 10.52 N is the cause of the horizontal acceleration that is measured.
2. The foot is decelerating its upward rise at the end of lift-off $\Rightarrow R_{yp}$ (3.69 N) is somewhat less than the static gravitational force (11.37 N)
3. $\tau = \text{positive} \Rightarrow$ dorsiflexor activity (Tibialis anterior, Extensor digitorum longus, Peroneus tertius, Extensor hallucis longus)
Most of this torque (1.04 out of 1.33) goes to horizontal acceleration of foot's centre of mass. Only 0.23 Nm goes to angular acceleration of the foot's low I_0 .

Example 3:

For the same instant in time, as in example 2, calculate the muscle torque and the reaction forces at the knee joint.

The leg segment is 43.5 cm long.

Video analysis gives the following information:

$$a_x = -0.03 \text{ m/s}^2$$

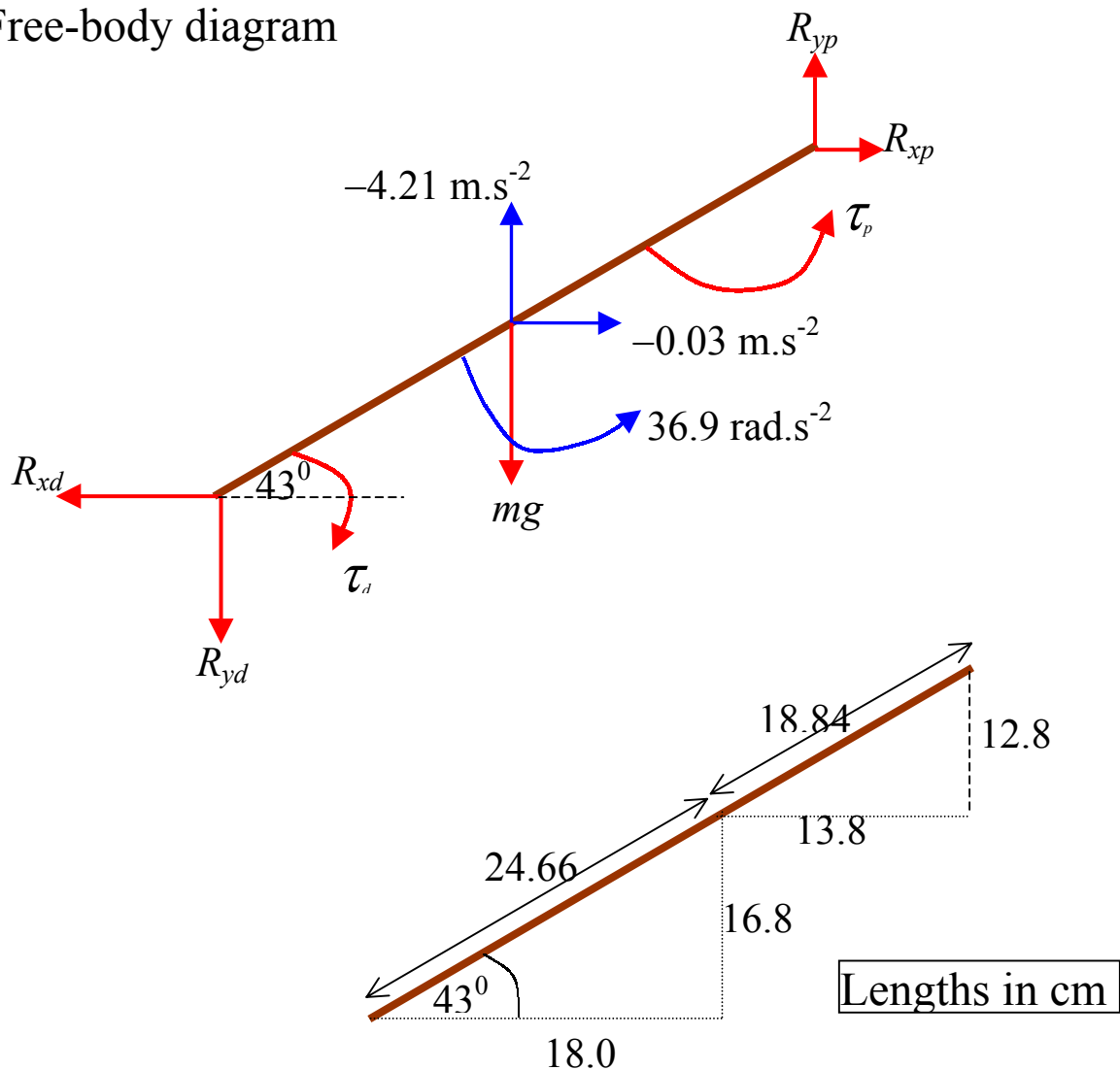
$$a_y = -4.21 \text{ m/s}^2$$

$$\alpha = 36.9 \text{ rad/s}^2$$

$$\theta = 43^\circ$$

Solution

Free-body diagram



Anthropometry:

Using anthropometrical tables:

$$m = 0.0465 \times 80 = 3.72 \text{ Kg}$$

$$\begin{aligned} \text{knee to centre of mass length} &= 0.433 \times 43.5 \text{ cm} \\ &= 18.84 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{centre of mass to ankle length} &= 43.5 - 18.84 \\ &= 24.66 \text{ cm} \end{aligned}$$

[Note; these two lengths are shown in the above diagram, where the moment arms of the joint reaction forces are also shown]

From Eg 2 we have

$$R_{xd} = 10.52 \text{ N}$$

$$R_{yd} = 3.69 \text{ N}$$

$$\tau_d = 1.33 \text{ N}$$

Newton's third law – note that in the free-body diagram the directional sense has been reversed after crossing the ankle.

$$1 - \sum F_x = ma_x$$

$$R_{xp} - R_{xd} = ma_x$$

$$\Rightarrow R_{xp} = R_{xd} + ma_x$$

$$R_{xp} = 10.52 + 3.72(-0.03) = 10.41 \text{ N}$$

$$2 - \sum F_y = ma_y$$

$$R_{yp} - R_{yd} - mg = ma_y$$

$$\begin{aligned} \rightarrow R_{yp} &= R_{yd} + mg + ma_y = R_{yd} + m(g + a_y) \\ &= 3.69 + 3.72(9.8 - 4.21) = 24.48 \text{ N} \end{aligned}$$

$$3 - \text{About the centre of mass of leg, } \sum \tau = I_0 \alpha$$

$$\Rightarrow \tau_p - \tau_d - 0.168R_{xd} + 0.180R_{yd} - 0.128R_{xp} + 0.138R_{yp} = I_0 \alpha$$

$$\begin{aligned} \Rightarrow \tau_p &= \tau_d + 0.168R_{xd} - 0.180R_{yd} + 0.128R_{xp} - 0.138R_{yp} + I_0 \alpha \\ &= 1.33 + 0.168 \times 10.52 - 0.180 \times 3.69 + 0.128 \times 10.41 \\ &\quad - 0.138 \times 24.48 + 0.0642 \times 36.9 \\ &= 1.33 + 1.77 - 0.664 + 1.33 - 3.38 + 2.37 \end{aligned}$$

$$\therefore \tau_p = 2.67 \text{ N.m}$$

Note:

1. τ_p is positive \Rightarrow a counterclockwise τ acting on knee
 \Rightarrow knee extensor dominates i.e. quads are rapidly extending the swinging leg.
2. The net angular acceleration of the leg is the net result of reaction forces and muscles torques at either end. There is no single primary force causing the movement observed – all play a significant role.

2.2 Force Transducer and force plates.

In order to measure the force exerted by the human body on an external body or load, we need a suitable measuring device. Such a device is called a force transducer. It gives an electrical signal proportional to the applied force e.g.

Strain gauge

Piezoelectric

Piezoresistive

Capacitive and others

All work on the principle that the applied force causes a certain amount of strain within the transducer.

2.2.1 Multidirectional Force Transducers.

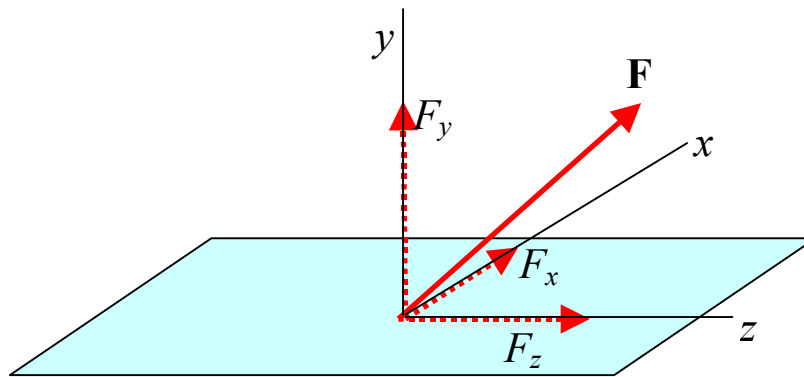
In order to measure forces in two or more directions it is necessary to use a bi or tri-directional force transducer. Such a device consists of 2 or 3 transducers mounted at right angles.

Need to ensure that the applied force acts through the central axis of each of the individual transducers.

2.2.2 Force plates.

Most common force acting on the human body is ground reaction force – acts during standing, walking, running.

This force vector is 3D – consisting of a vertical component and 2 shear components in horizontal plane



The shear components are usually resolved into

anterior – posterior (x)

and

medial – lateral (z)

What else do we need?

We need the location of the centre of pressure!

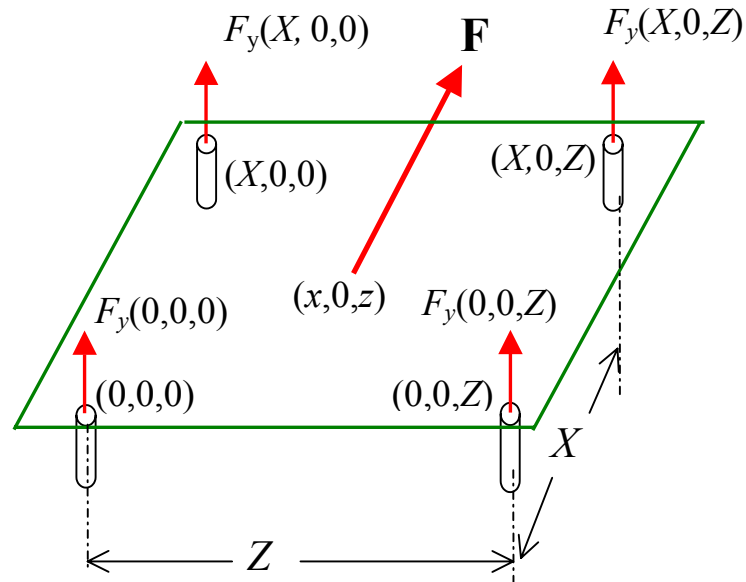
The foot is supported over a varying surface area, with different pressures at each part – we would be faced with the expensive problem calculating the net effect of all these pressures as they change with time.

For most applications it is sufficient to know the centre of pressure only.

A force plate can give us F_x , F_y , F_z and centre of pressure – necessary to complete the inverse solution.

There are two common types of plates:

1. A flat plate supported by four triaxial transducers.



The four transducers are located at $(0,0,0)$, $(X,0,0)$, $(0,0,Z)$ and $(X,0,Z)$.

x – component of \mathbf{F} :

$$F_x = F_x(0,0,0) + F_x(X,0,0) + F_x(0,0,Z) + F_x(X,0,Z)$$

y – component \mathbf{F} :

$$F_y = F_y(0,0,0) + F_y(X,0,0) + F_y(0,0,Z) + F_y(X,0,Z)$$

z – component \mathbf{F} :

$$F_z = F_z(0,0,0) + F_z(X,0,0) + F_z(0,0,Z) + F_z(X,0,Z)$$

Thus we have

$$\mathbf{F} = (F_x, F_y, F_z)$$

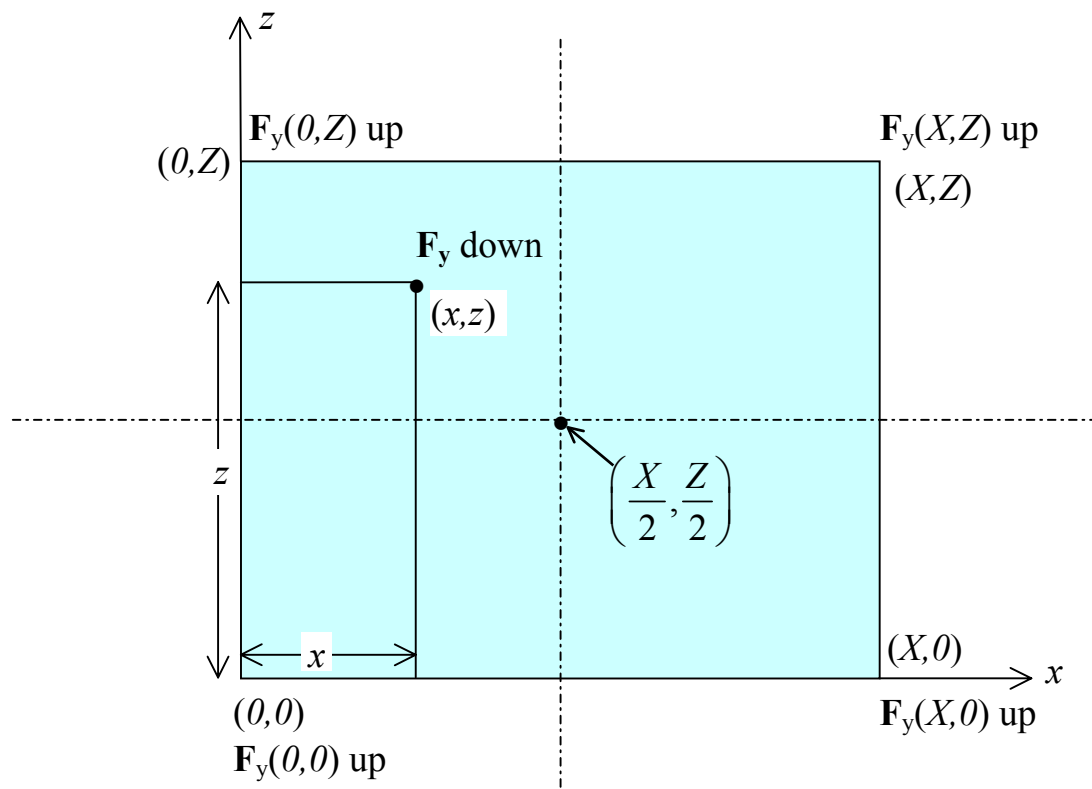
How do we find centre of pressure? i.e. how do we determine $(x,0,z)$?

Suppose all four F_y 's were equal?

$\Rightarrow (x,0,z)$ was at the exact centre of the plate.

i.e. $x=X/2$

$z=Z/2$



In general,

$$x = \frac{X}{2} \left[1 + \frac{(F_y(X,0,0) + F_y(X,0,Z)) - (F_y(0,0,0) + F_y(0,0,Z))}{F_y} \right]$$

$$z = \frac{Z}{2} \left[1 + \frac{(F_y(0,0,Z) + F_y(X,0,Z)) - (F_y(0,0,0) + F_y(X,0,0))}{F_y} \right]$$

Exercise:

Derive these expressions. Hint: revise “determination of centre of mass in 2D” this time taking axes A and B through mid-point of force plate and parallel to x and z axes.

Why do we require the 4 transducers instead of 3 transducers plus 1 support?

2. Central support force plate

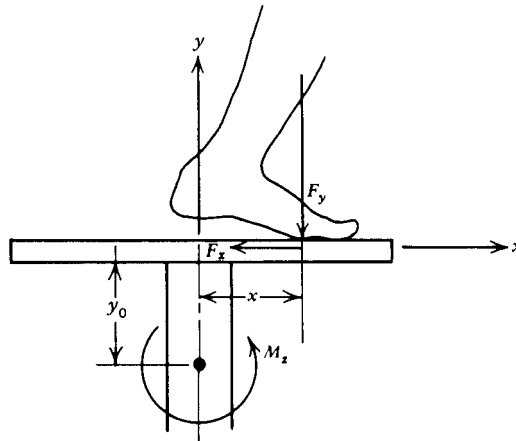


Figure 4.10 Central support type force plate, showing the location of the center of pressure of the foot and the forces and moments involved.

One centrally instrumental pillar supporting an upper flat plate.

The action force of the foot F_y acts downward.

F_x is the anterior-posterior shear, either forward or backward (we have shown it backward)

$\sum \tau = 0$ about central axis of support

$$\Rightarrow \tau_z - F_y x + F_x y_0 = 0$$

$$\Rightarrow x = \frac{F_x y_0 + \tau_z}{F_y}$$

τ_z = torque (bending moment) about axis of rotation of support

y_0 = distance from support axis to force plate surface.

F_x , F_y and τ_z (all measured) continuously change with time $\Rightarrow x$ can be calculated to show centre of pressure moving across force plate.

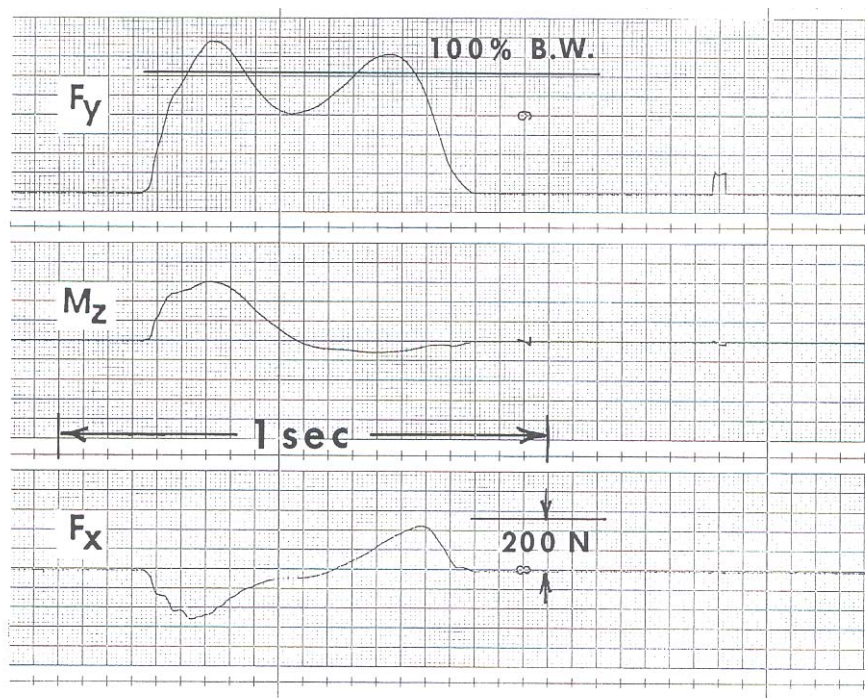


Figure 4.11 Force plate record obtained during gait, using a central support type as shown in Figure 4.10.

The above figure shows typical force plate data for a subject walking at normal speed.

As the heel strikes the plate:

- F_y (vertical reaction force) rapidly rises to a value in excess of body weight.
- F_x is negative (backward) – if not the foot would slide forward i.e. on ice.
- τ_z is positive (anti-clockwise)

As the knee flexes during midstance:

- F_y drops to below body weight
- F_x is small
- τ_z is small

At push off:

- F_y again rises above body weight – body is being accelerated up as the plantar flexors become active.
- F_x is positive – accelerating the body forward – if not the foot would slip backward.
- τ_z is slightly negative (clockwise)

Throughout the movement the centre of pressure will move forward from the heel to toes relative to the foot.

Note: The reaction force components are the algebraic sums of all mass \times acceleration products of all body segments.

So for an N–segment system this \Rightarrow :

$$1. \quad F_x = \sum_{i=1}^N m_i a_{xi}$$

where $F_x = x$ -component of reaction force

$a_{xi} =$ acceleration of i^{th} -segment centre of mass
in the x direction.

$$2. \quad F_y - Mg = \sum_i m_i a_{yi}$$

but

$$Mg = \sum_i m_i g$$

thus

$$F_y - \sum_i m_i g = \sum_i m_i a_{yi}$$

therefore

$$F_y = \sum_{i=1}^N m_i (a_{yi} + g)$$

Where $a_{yi} =$ acceleration of i^{th} -segment centre of mass
in y -direction.

$g =$ magnitude of acceleration due to gravity.

When a person stands perfectly still on a plate

$$F_y = \sum_{i=1}^N m_i g = Mg.$$

If $F_y = Mg$

Then is the person perfectly still? That is, are all body
parts stationary?

No! (not necessarily)

The person could be accelerating one arm up at the same rate they are accelerating the other down.

→ Interpretation of ground reaction forces as far as what individual segments are doing is virtually impossible.

- One ground reaction force may be due to a number of different motions of the segments.
- Combination of general reaction forces and kinematic data allows a “complete” description of motion.
- Force plate data and kinematic data (video) must be synchronized! (They came from completely separate systems).

2.3.3 Combined Force plate and kinematic data.

It is necessary to combine data from force plates with segment kinematics to calculate muscle torques and reaction forces at joints.

Example:

Consider the ankle joint during dynamic stance.

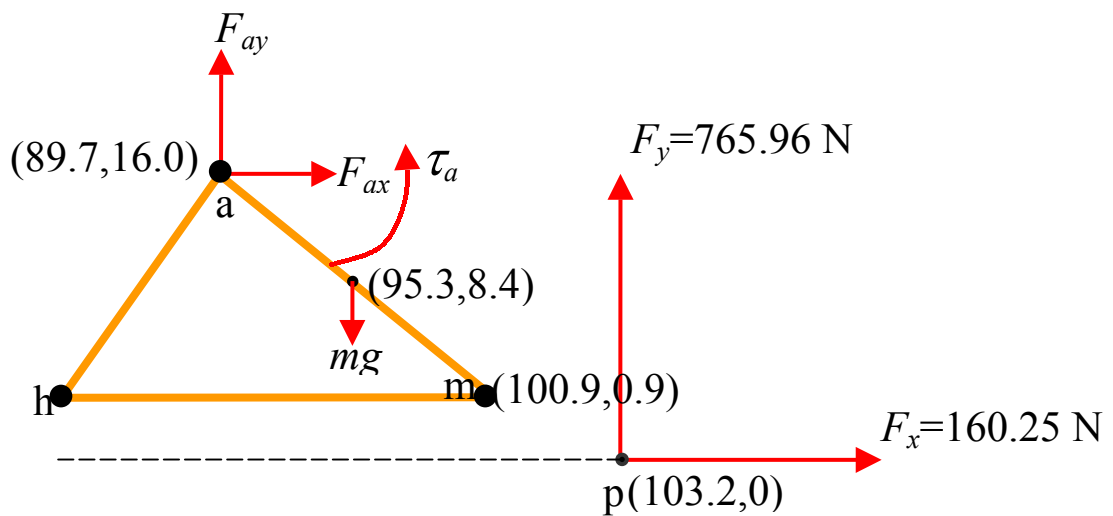
A subject in the push off stance records the following kinematic data:

$$a_x = 3.25 \text{ m/s}^2$$

$$a_y = 1.78 \text{ m/s}^2$$

$$\alpha = -45.35 \text{ rad/s}^2$$

The free-body diagram and necessary anthropometry are set out below.



$$m_{foot} = 1.12 \text{ Kg}$$

$$I_0 = 0.01 \text{ Kg.m}^2$$

Other data is shown on the diagram.

a = ankle at position (89.7, 16.0) cm

m = metatarsal at position (100.9, 0.9) cm

p = centre of pressure at position (103.2, 0.0) cm

h = heel, position not required for the problem.

Solution: (Working from the above free-body diagram)

$$1. \sum F_x = ma_x$$

$$\Rightarrow F_{ax} + F_x = ma_x$$

$$\begin{aligned} \Rightarrow F_{ax} &= ma_x - F_x \\ &= 1.12 \times 3.25 - 160.25 \\ &= -156.6 \text{ N} \end{aligned}$$

$$2. \sum F_y = ma_y$$

$$\Rightarrow F_{ay} - mg + F_y = ma_y$$

$$\begin{aligned} \Rightarrow F_{ay} &= m(a_y + g) - F_y \\ &= 1.12(1.78 + 9.81) - 765.96 \\ &= -753.0 \text{ N} \end{aligned}$$

$$3. \sum \tau = I_0 \alpha \text{ (about centre of mass)}$$

$$\begin{aligned} \Rightarrow \tau_a + F_x(0.084 - 0) + F_y(1.032 - 0.953) \\ - F_{ax}(0.160 - 0.084) - F_{ay}(0.953 - 0.897) &= I_0 \alpha \\ \Rightarrow \tau_a + F_x \times 0.084 + F_y \times 0.079 - F_{ax} \times 0.076 - F_{ay} \times 0.056 \\ &= 0.01 \times (-45.35) \\ \Rightarrow \tau_a &= -0.01 \times 45.35 - 160.25 \times 0.084 - 765.96 \times 0.079 \\ &\quad - 156.6 \times 0.076 - 753.0 \times 0.056 \\ &= -128.5 \text{ N.m} \end{aligned}$$

Ex

Draw “correct” diagram and add interpretation.

2.2.4 Interpretation of torque curves

Consider the torques about the ankle, knee and hip joints of a patient fitted with a total hip replacement.

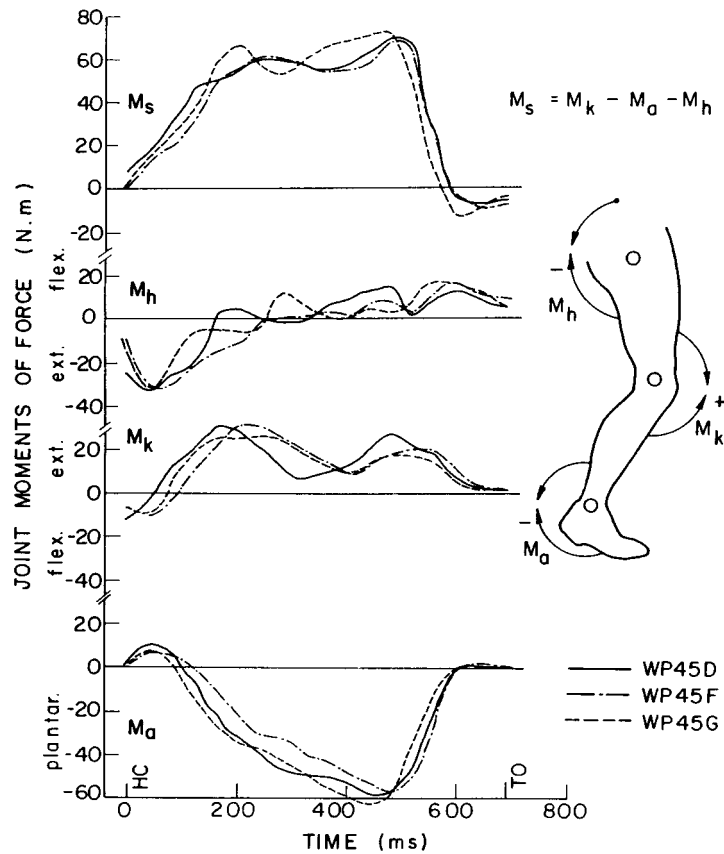


Figure 4.13 Joint moment-of-force profiles from three repeat trials of a patient fitted with a total hip replacement. See text for detailed discussion.

The plot shows 3 repeat trials.

Recall the convention for torque – positive torques are anticlockwise, negative torques are clockwise.

Thus:

Plantar flexor torque is negative

Knee extensor torque is positive

Hip extensor torque is negative

Consider the plots. The torques are plotted during stance
– from heel contact at $t = 0$ to toe-off at $t = 680$ ms.

1. Ankle joint

- First 80 ms – ankle generates positive τ as the dorsiflexors act eccentrically to lower the foot to the ground.
- Then the plantarflexors start to act \Rightarrow negative τ .
- In mid-stance they act to control the amount of forward rotation of the leg over the foot, which is flat on the ground.
- In late-stance they peak in order to produce “push-off”. This peak is usually deeper in normals – but is reduced in this case because of the pathology related to hip replacement.
- Just before toe-off $\tau \rightarrow 0$ – the limb is said to be unloaded as the other foot now bears body weight. For the last 90 ms the toe is just touching the force plate.

2. Knee joint

- The knee extensors are effectively active during all stance \Rightarrow positive τ .
- A little bit of knee flexor activity is evident just as the heel strikes \Rightarrow eccentric action of hamstrings in slowing down knee extension around heel strike.

- Quads are active in early stance to control the amount of knee flexion as leg supports body weight.
- In mid-stance they act to extend the knee
- During push-off the knee starts flexing in preparation for swing; the quads act eccentrically to control the amount of knee flexion.

3. Hip joint

- At heel contact, τ_h is negative \Rightarrow extensors are active.
This remains the case until \sim mid-stance. This has two functions:
 1. The hip extensors act on the thigh to assist quads in controlling knee flexion
 2. The hip extensors act to control forward rotation of the upper body as it attempts to rotate forward over the hip joints.
- In the latter half of stance τ_h is positive \Rightarrow hip flexors are active to:
 1. Initially reverse backward rotation of thigh
 2. Pull the thigh forward & upward – occurs around the time of toe-off.

Top curve

$$\tau_s = \tau_k - \tau_a - \tau_h \text{ (support torque)}$$

$$= \text{sum of extensor torques}$$

Called support torque because it represents a total limb pattern to push away from the ground.

2.2.5 Differences between centre of mass and centre of pressure.

Centre of mass was defined in the section “Centre of mass of a multi-segment system” – it’s the weighted average of the centre of mass of each body segment

$$(x, y) = \frac{1}{M} \sum_i m_i (x_i, y_i)$$

It depends upon the relative mass of each segment (m_i/M) and its relative position (x_i, y_i) – it is independent of velocity and acceleration.

Centre of pressure is quite independent of centre of mass. It is the location of the ground reaction force. It is equal and opposite to a weighted average of the locations of all downward (action) forces acting on the ground.

These forces are under the motor control of the ankle muscles. The centre of pressure is the neuromuscular response to imbalances of the bodies centre of mass.

Consider a person standing still on a force plate

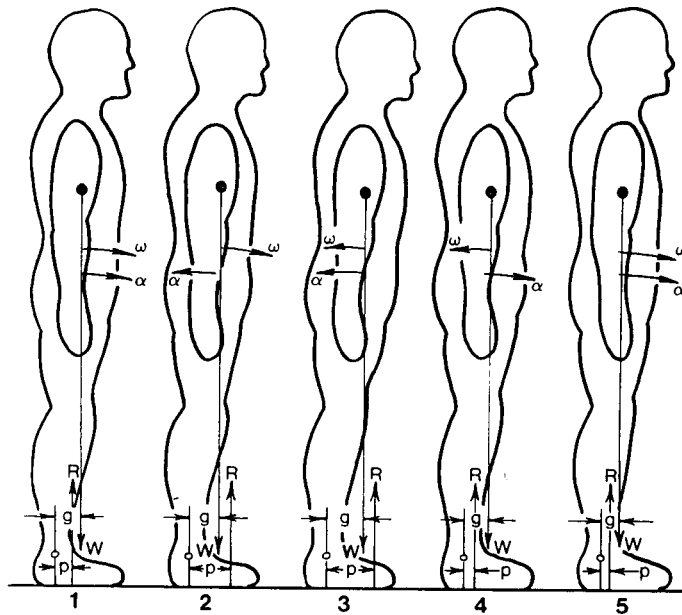


Figure 4.15 A subject swaying back and forth while standing on a force platform. Five different points in time are described, showing the center of gravity and the center of pressure locations along with the associated angular accelerations and velocities of the body. See text for detailed description.

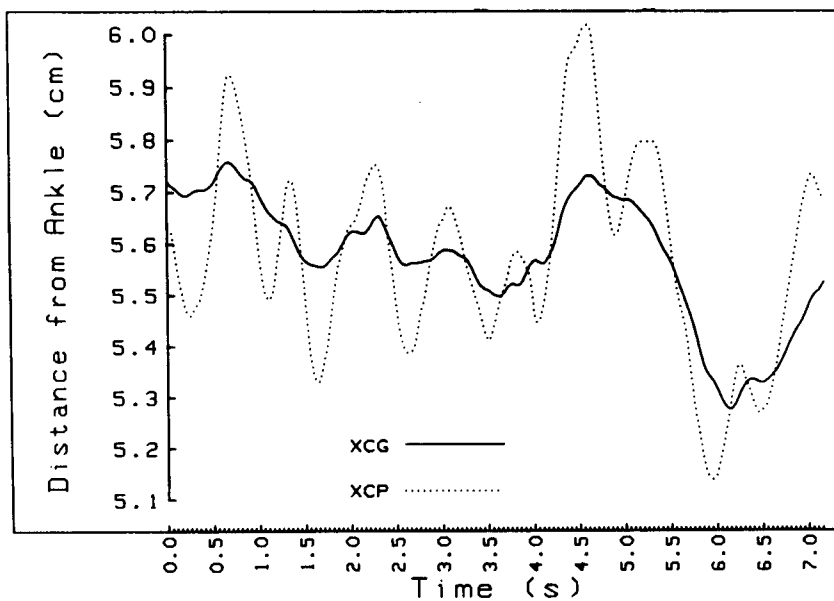


Figure 4.16 A 7-s record showing simultaneous center of gravity and center of pressure plots for a subject in quiet stance. Center of pressure excursions are higher frequency and have a greater amplitude.

Think of the body as an inverted pendulum, pivoting about the ankle. The centre of pressure moves in response to movement of centre of mass in an attempt to establish equilibrium.

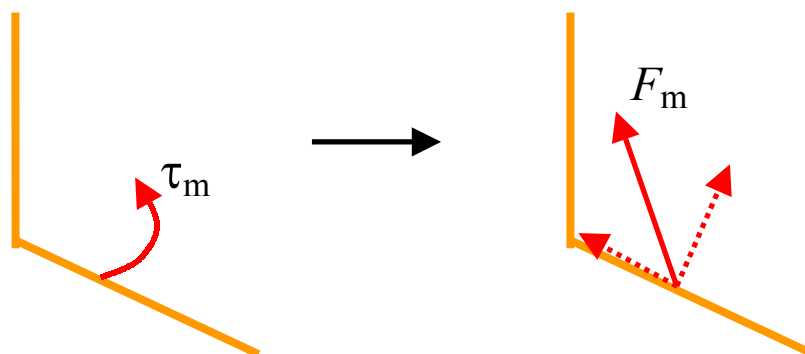
What happens if the centre of mass moves beyond the toes? – you are forced to move a limb.

2.3 Bone-on-bone forces during dynamic conditions.

- Link-segment models assumes:
 1. A hinge joint.
 2. Muscle torque is generated by a torque motor.

⇒ Reaction force across joint is the same as bone-on-bone force at joint.
- But muscles are linear motors not torque motors.

⇒ The presence of additional compressive and shear forces across the joint surfaces.
- Thus in a more rigorous analysis the free body diagram should include these additional muscle-induced forces.



- In addition at the extreme range of joint movement force from ligaments and other anatomical constraints become important; we continue to ignore these.

2.3.1 Indeterminacy in muscle force estimates.

- Estimating muscle force is a major problem – even with good estimates of muscle torques at each joint.
- The solution is essentially indeterminate – more unknowns than there are equations.
- A knowledge of the tension in individual muscle, as a function of time, during human movement would be of considerable value.

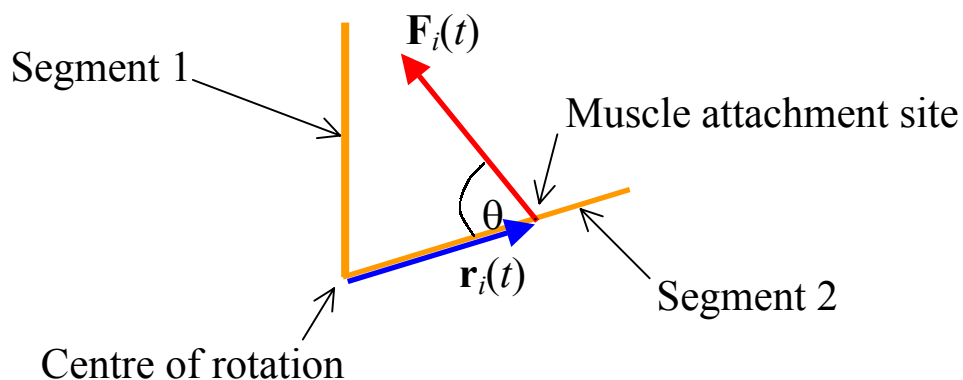
The torque generated at any time t , by a muscle i crossing a joint j is given by:

$$\tau_i(t) = \mathbf{r}_i(t) \times \mathbf{F}_i(t).$$

Where

$\mathbf{r}_i(t)$ = displacement vector drawn from the centre of rotation to the attachment site of muscle i .

$\mathbf{F}_i(t)$ = tension (vector) in muscle i .



If

$$d_i(t) = r_i(t) \sin(\theta(t)) - \text{moment arm of } \mathbf{F}_i(t)$$

then

$$\begin{aligned} \tau_i(t) &= F_i(t) d_i(t) \\ &= \text{magnitude of torque due to muscle } i \text{ at any} \\ &\quad \text{time } t. \end{aligned}$$

If we consider motion in a plane only, then the net muscle torque about a joint is the algebraic sum of all anticlockwise torques minus the algebraic sum of all clockwise torques, i.e.

$$\tau_j = \sum_{i=1}^{N_a} F_{ai}(t) d_{ai}(t) - \sum_{i=1}^{N_c} F_{ci}(t) d_{ci}(t) \quad - (*)$$

Where:

N_a = Number of muscles producing anticlockwise torque.

N_c = Number of muscles producing clockwise torque.

$F_{ai}(t)$ = Force of i^{th} muscle producing anticlockwise τ at t .

$F_{ci}(t)$ = Force of i^{th} muscle producing clockwise τ at t .

$d_{ai}(t)$ = Moment arm of i^{th} muscle producing anticlockwise τ at t .

$d_{ci}(t)$ = Moment arm of i^{th} muscle producing clockwise τ at t .

Example:

For the knee, the joint is the knee joint, so

$$j \rightarrow k$$

The knee extensors produce anticlockwise torque, so

$$a \rightarrow e$$

The knee flexors produce clockwise torque, so

$$c \rightarrow f$$

Therefore equation (*) may be written as

$$\tau_k = \sum_{i=1}^{N_e} F_{ei}(t) d_{ei}(t) - \sum_{i=1}^{N_f} F_{fi}(t) d_{fi}(t)$$

N_e = number of knee extensors

N_f = number of knee flexors

Example:

Consider the torque being produced about the ankle during late stance.

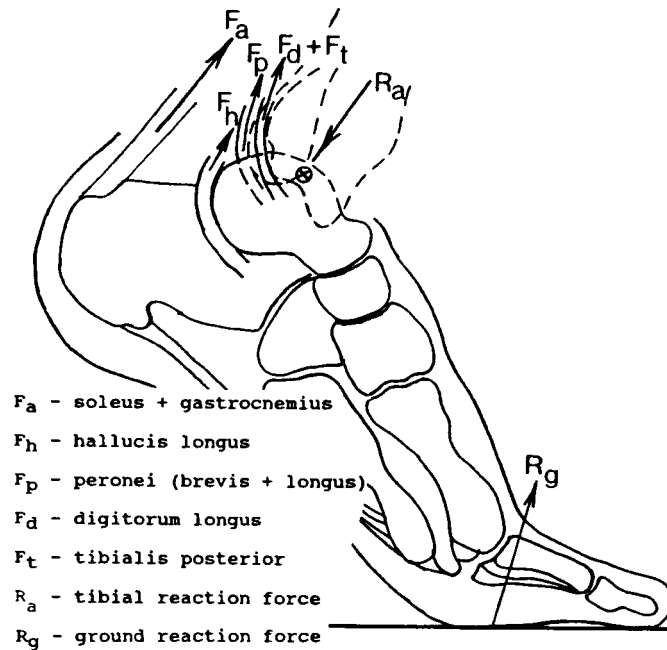


Figure 4.18 Anatomical drawing of foot and ankle during the push-off phase of a runner. The tendons for the major plantarflexors as they cross the ankle joint are shown along with ankle and ground reaction forces.

Formally the net ankle torque (which comes from an inverse solution) is given by,

$$\tau_a(t) = \sum_{i=1}^{N_d} F_{di}(t) d_{di}(t) - \sum_{i=1}^{N_p} F_{pi}(t) d_{pi}(t) \quad - (\dagger)$$

Where

N_d = number of dorsiflexors

N_p = number of plantarflexors

In this equation

$\tau_a(t)$ is known – comes from inverse solution.

d_{di} and d_{pi} are known – assume we can get these moment arms from muscle anthropometry.

But the tension (force) in the dorsiflexors and plantarflexors is unknown.

So we have one equation in about 12 unknowns (4 dorsiflexors and 8 plantarflexors)

This problem is indeterminate! How do we solve it?

The indeterminacy problem is solved by assuming:

1. No co-contraction of agonists and antagonists
2. The stress of each active muscle is equal.

In the present example assumption **1** means that there is no co-contraction of plantarflexors and dorsiflexors. Since the foot is in late stance, the plantarflexors are active in producing plantar flexion, hence this assumption

$\Rightarrow F_{di}(t) = 0$ for all i , i.e. for all dorsiflexors.

Thus equation (†) reduces to,

$$\tau_a = -\sum_{i=1}^{N_p} F_{pi}(t) d_{pi}(t) \quad - (\#)$$

Recall that the tension in a muscle can be written as,

$$F = PCA \times S$$

Where,

PCA = physiologic (functional) cross-sectional area

S = muscle stress (force per cross-sectional area)

[See section 1 – muscle anthropometry]

Assumption **2** says that S is the same for all muscles, therefore,

$$F_i = PCA_i \times S$$

Where,

F_i = tension in muscle i

PCA_i = functional cross-sectional area of muscle i

With this assumption equation (#) reduces to,

$$\tau_a = -\sum_{i=1}^{N_p} PCA_{pi} S(t) d_{pi}(t)$$

Where,

PCA_{pi} = functional cross-sectional area of the i^{th}

Plantarflexor and

$$F_{pi}(t) = PCA_{pi} S(t) . \quad - (\dagger\dagger)$$

So

$$\tau_a(t) = -S(t) \sum_{i=1}^{N_p} PCA_{pi} d_{pi}(t) \quad - \text{(\#\#)}$$

Both PCA_{pi} and d_{pi} come from muscle anthropometry, there equation (\#\#) is solved for $S(t)$

$$\Rightarrow S(t) = \frac{-\tau_a(t)}{\sum_{i=1}^{N_p} PCA_{pi} d_{pi}(t)}$$

With the numerator of the above solution coming from the inverse solution and the denominator coming from muscle anthropometry.

Having solved for $S(t)$, we can then use $S(t)$ in equation (\dagger\dagger) to calculate the tension in any of the plantarflexors ($F_{pi}(t)$) at time t in the movement.

3. Mechanical work, energy and power

3.0 Introduction

- In any biomechanical analysis, energetics is an important aspect.
- Without energy flows movement is not possible.
- Mechanical work calculations are essential in efficiency assessment of sport & work related tasks.
- Joint mechanical power is important in assessment of human motion.

We need to have certain terms & laws related to mechanical energy, work & power clearly in mind.

3.0.1 Mechanical Energy and Work

- Mechanical energy and work have the same units (joules).
- Mechanical energy is a measure of the state of a body at an instant in time as to its ability to do work.

E.g. A body with 200 J of kinetic energy & 150 J of potential energy is capable of doing 350 J of work (on another body).

- Work is the measure of energy flow from one body to another & time must elapse for that work to be done. If the energy flows from body A to body B, we say body A does work on body B.

Example:

If A is a muscle and B is a segment, then muscle A can do work on segment B if energy flows from the muscle to the segment.

3.0.2 Law of conservation of energy

Change in energy of a system = work done on system

$$\Delta E = \sum_i \Delta E_i = W \quad (3.1)$$

The sum is over all forms of energy in the system.

At all points in the body & at all instances in time the law of conservation of energy applies.

Example:

Any body will change its energy only if there is a flow of energy into or out of any adjacent structure (tendons, ligaments or joint contact surfaces).

For a body segment equation (3.1) can be written as,

$$\Delta E_{segment} = \text{net flow of energy into segment} \quad (3.2)$$

Example:

Consider a segment which is in contact at the proximal and distal ends & has 4 muscle attachments. With this arrangement there are 6 possible routes for energy flow. The various flows are as shown.

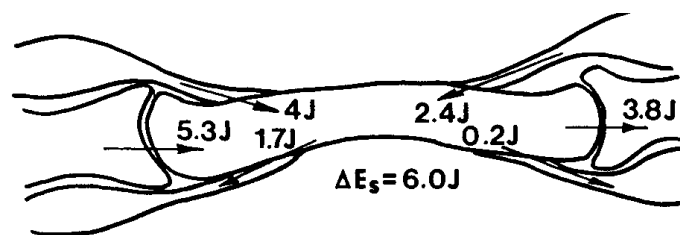


Figure 5.1 (a) Flow of energy into and out of a segment from the adjacent connective tissue and joint contacts over a period of time Δt .

What is the energy change of the segment?

Conservation of energy ((3.2)) \Rightarrow

$$\Delta E_s = 4 + 5.3 + 2.4 - 1.7 - 0.2 - 3.8 = 6.0J$$

If this flow occurs over a time period of $20ms$, what is the power flow into the segment?

Recall, power is the rate of doing work, i.e.

$$\begin{aligned} P_s &= \frac{\Delta E_s}{\Delta t} \\ &= \frac{4 + 5.3 + 2.4 - 1.7 - 0.2 - 3.8}{0.02} \\ &= \frac{6.0}{0.02} = 300W \end{aligned}$$

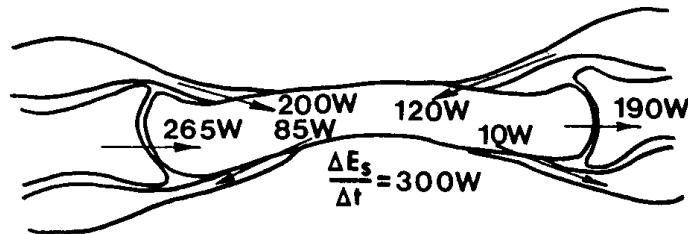


Figure 5.1 (b) Rate of flow of energy (power) for the same segment and same point in time as (a). A “power balance” can be calculated; see text for discussion.

Energy conservation → energy storage within segment.

Energy storage within segments takes the form of potential and kinetic energy (both translational and rotational).

Thus E_s at any time = $PE + KE$, irrespectively of energy flows into and out of the segment.

3.0.3 Internal and external work

- Muscles are the only source of mechanical energy generation in the body. They are also a major site of energy absorption.
- Only a very small fraction of energy is dissipated into heat as a result of joint friction and viscosity in the connective tissue.
- Mechanical energy is continuously flowing into and out of muscles and from segment to segment. To reach an external load, many energy changes may occur in the segments between the source and external load.

Example:

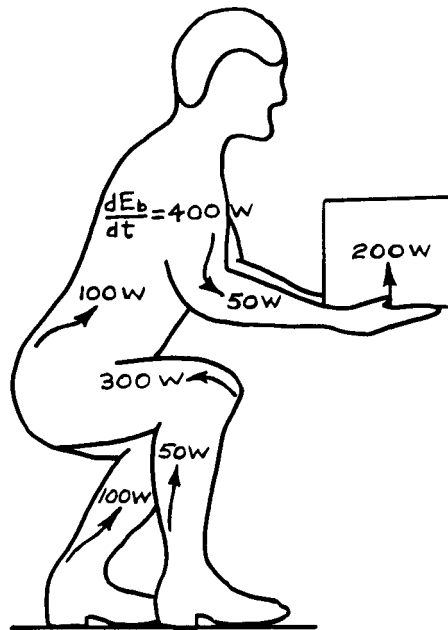


Figure 5.2 Lifting task showing the power generation from a number of muscles and the combined rate of change of energy of the body (internal work) and rate of energy flow to the load (external work).

The work rate in lifting an external load may be $200W$.

This may however require an additional $400W$ of power by source muscles to perform the task.

So,

$$W_{internal} + W_{external} = 600W$$

Where

$W_{internal}$ = changing Mechanical energy of body segments

$W_{external}$ = work done lifting external load

- For movements like walking and running, there is no external load – all energy generation and absorption is required to move the body segments themselves.

- A distinction is made between work done on body segments and work done on external loads.
Work done on body segments is called internal work
Work done on external loads is called external work
Lifting weights, pushing a car and cycling on an ergometer have well-defined loads.
- There is one exception to the external work definition:
 - lifting one's body weight to a new height.
Thus running up a hill involves both internal and external work.
- External work can be negative if an external force is exerted on the body and the body gives way.
E.g. tackling, catching a ball.
Can internal work be negative?

3.1 Efficiency

The term efficiency is problematic when applied to human movement energetics.

Metabolic energy is converted to mechanical energy at the tendons.

Metabolic efficiency depends upon:

1. Condition of each muscle.
2. Metabolic state of muscle (fatigue).

3. Subject's diet.

4. Any possible metabolic disorder.

A definition of metabolic or muscle efficiency would be;

$$\text{Metabolic (muscle) efficiency} = \frac{\sum \text{mechanical work done by all muscles}}{\text{metabolic energy consumption of muscles}}$$

This is impossible to calculate because:

1. Can't calculate work of each muscle

2. Can't isolate metabolic energy of muscle

i.e. neither the numerator nor the denominator are known!

With this in mind we could define a mechanical efficiency

as

$$\text{Mechanical efficiency} = \frac{\text{Mechanical work (internal + external)}}{\text{metabolic cost} - \text{resting metabolic cost}}$$

Resting metabolic cost in cycling, for example, could be the cost associated with sitting still on the bicycle.

Internal work? Can it be determined readily?

A definition of work efficiency is:

$$\text{Work efficiency} = \frac{\text{External mechanical work}}{\text{metabolic cost} - \text{zero work metabolic cost}}$$

In the cycling example, zero-work cost would be the cost measured with the cyclist freewheeling.

For equal levels of work (external);

The metabolic cost of positive work $>$ metabolic cost of negative work.

However negative W is not (usually) negligible.

Level gait has equal amount of positive and negative W ,
i.e.

$$W_+ = W_-$$

$$\text{Uphill gait} \rightarrow W_+ > W_-$$

$$\text{Downhill gait} \rightarrow W_+ < W_-$$

In general human movements have varying amounts of positive and negative work, and since the metabolic cost of positive and negative are different

\Rightarrow efficiency calculations yield numbers that are strongly influenced by the relative percentages of positive and negative work.

Attempts to get around this problem involve ‘splitting’ the efficiency into ‘positive’ and ‘negative’ work components.

Metabolic cost of W_+ + metabolic cost of W_- = metabolic cost.

Define

$$e_+ = W_+ / (\text{metabolic cost of } W_+)$$

$$e_- = W_- / (\text{metabolic cost of } W_-)$$

$$\Rightarrow \frac{W_+}{e_+} + \frac{W_-}{e_-} = \text{metabolic cost}$$

if $W_+ = W_-$, then $e_- > e_+$.

Example:

Application of the work-energy principle for running with and without shoes.

The consumption of metabolic energy during physical activity can be quantified by measuring O_2 consumption of the athlete during the activity of interest.

Measurement of O_2 consumption for running barefoot and running with shoes typically shows about 5% in favor of barefoot running.

→ running barefoot requires less O_2 and therefore less energy.

⇒ More efficient? – cover same distance with less energy expenditure.

A 5% energy gain is substantial – if this energy saving can be transferred to a time saving

⇒ running barefoot would gain about 6 –7 minutes for the marathon and about 0.5 seconds for the 100 meter sprint.

∴ worthy of a closer look.

Questions to be answered.

Determine the additional mechanical work that an athlete wearing shoes must do,

1. Against gravity
2. To accelerate the additional shoe mass.

Assumptions:

1. Metabolic cost of marathon runner $\sim 10MJ$
2. Mass of shoe = 100g (light)
3. Each foot (and shoe) is lifted during each step by
 $\Delta h = 0.2m$
4. The maximal speed of the swing leg during the swing phase is 10m/s (middle – long distance running)
5. Step length (left toe – right toe) is 2m.
 $n \sim 20,000$ steps during marathon.

Solution:

Additional work against gravity.

$$\text{Extra work per stride} = \Delta mg \Delta h$$

$$\rightarrow \Delta W_{gr} = n \Delta mg \Delta h$$

$$= 20,000 \times 0.1Kg \times 10m/s^2 \times 0.2m$$

$$= 4,000J$$

Additional work done in the acceleration of shoe mass.

Work done per stride in acceleration shoe = ΔKE of shoe.

$$\Delta KE = \frac{1}{2} \Delta m v^2$$

Therefore total additional work done in accelerating shoe is,

$$\begin{aligned} \Delta W_{acc} &= n \frac{1}{2} \Delta m v^2 \\ &= 20,000 \times \frac{1}{2} \times 0.1 \text{Kg} \times (10 \text{m/s})^2 \\ &= 100,000 \text{J} \end{aligned}$$

\Rightarrow Total extra work $\sim 10^5 \text{J}$

$\Rightarrow \frac{\text{Extra work}}{\text{Metabolic cost}} = \frac{10^5}{10^7} = 1\% \text{ increase}$

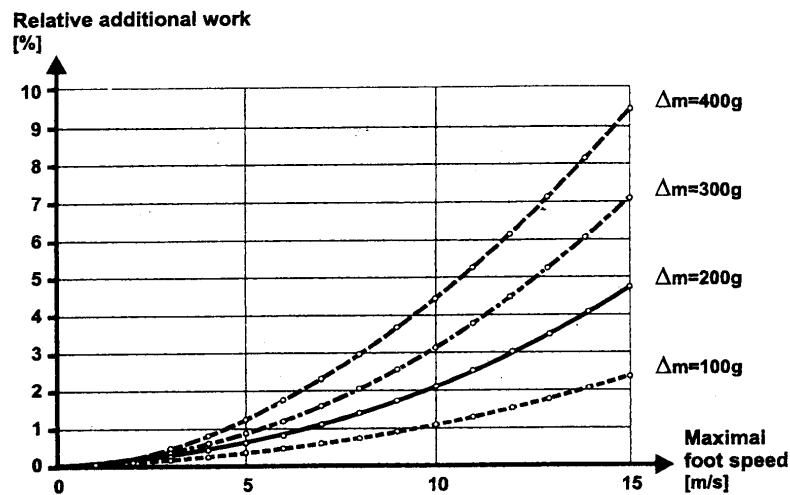


Figure 1.2 Relative additional mechanical work of an airborne foot due to the acceleration of an additional shoe mass as a function of the maximal speed for running.

3.1.1 Positive work of muscles.

Positive work is done during a concentric contraction, when the muscle torque acts in the same direction as the angular velocity of the segment.

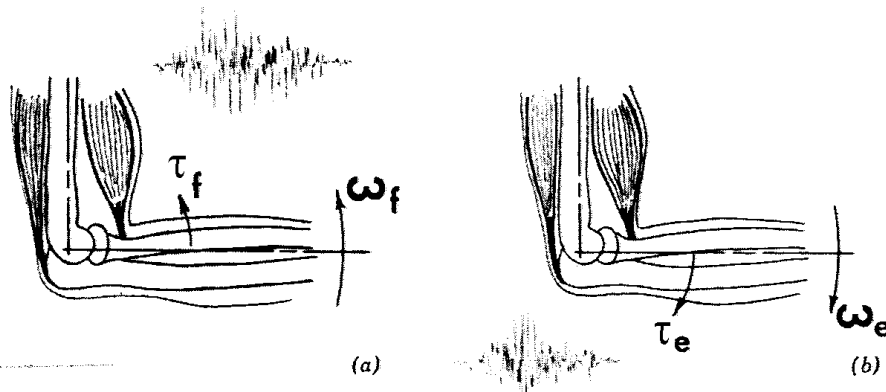


Figure 5.4 Positive power as defined by the net muscle moment and angular velocity. (a) A flexion moment acts while the forearm is flexing. (b) An extension moment acts during and extensor angular velocity. (Reproduced by permission of *Physiotherapy Canada*.)

If

$$\tau_m = +ve \text{ and } \omega_s = +ve$$

then

$$P_m = \tau_m \omega_s = +ve \Rightarrow +ve W$$

If

$$\tau_m = -ve \text{ \& } \omega_s = -ve$$

then

$$P_m = \tau_m \omega_s = +ve \Rightarrow +ve W$$

3.1.2 Negative work of muscles

Negative work is done during eccentric contraction, when muscle torque acts in the opposite direction to angular velocity of the segment.

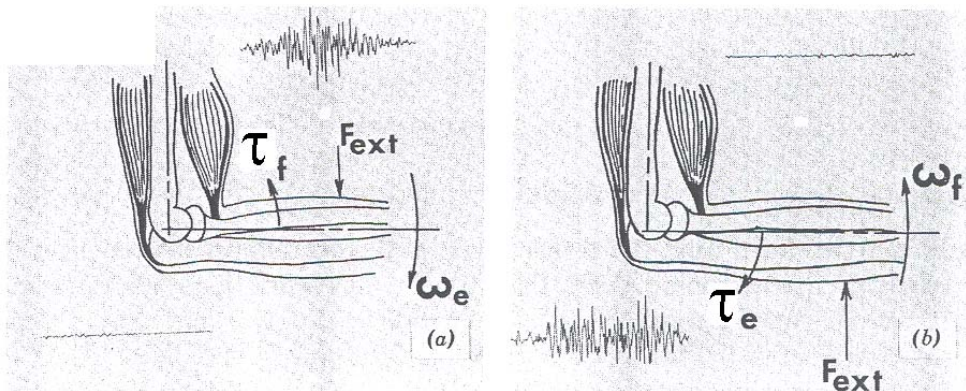


Figure 5.5 Negative power as defined by net muscle moment and angular velocity. (a) An external force causes extension when the flexors are active. (b) An external force causes flexion in the presence of an extensor muscle moment. (Reproduced by permission of *Physiotherapy Canada*.)

This usually happens when an external force F_{ext} acts on the segment and creates a joint torque $> \tau_m$.

If

$$\tau_m = +ve \text{ and } \omega_s = -ve$$

then

$$P_m = \tau_m \omega_s = -ve \Rightarrow -ve W$$

If

$$\tau_m = -ve \text{ \& } \omega_s = +ve$$

then

$$P_m = \tau_m \omega_s = -ve \Rightarrow -ve W$$

3.1.3 Muscle mechanical power.

- Rate of work done by most muscles is rarely constant with time.
- At a given joint, muscle power is the product of net muscle moment and angular velocity.

$$P_m = \tau_j \omega_j$$

Where

P_m = muscle power

τ_j = net muscle torque about joint j

ω_j = joint angular velocity

As already discussed P_m = positive or negative. Even during simple movements, P_m may reverse sign several times.

Example:

Consider extension and flexion of the forearm

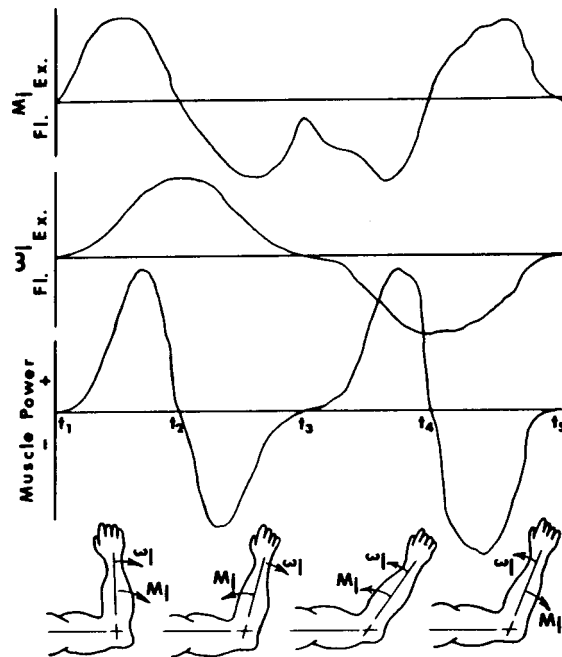


Figure 5.6 Sequence of events during simple extension and flexion of forearm. Muscle power shows two positive bursts alternating with two negative bursts.

Note:

t_j and $\omega_j \sim 90^\circ$ out of phase.

$t_1 - t_2$ triceps do +ve work – they accelerate extension.

$t_2 - t_3$ biceps do –ve work – they decelerate extension.

$t_3 - t_4$ biceps do +ve work – they accelerate flexion.

$t_4 - t_5$ triceps do –ve work – they accelerate flexion.

3.1.4 Mechanical work of muscles.

Power = rate of doing work

$$P = \frac{dW}{dt}$$

Thus to determine work from power we must integrate power w.r.t time, over the time period of interest.

$$dW = P(t)dt$$

$$\rightarrow W = \int_{t_i}^{t_f} P(t)dt$$

Where,

W = work done during time period t_i to t_f .

In the above example the work done during the period t_1 to t_2 is,

$$W_m = \int_{t_1}^{t_2} P_m dt = \text{area under power curve between } t_1 \text{ and } t_2.$$

Work done from t_1 to t_2 is positive.

Work done from t_2 to t_3 is negative.

Work done from t_3 to t_4 is positive.

Work done from t_4 to t_5 is negative.

If the forearm returns to the starting position, then the net mechanical work is zero meaning,

$$\int_{t_1}^{t_5} P_m(t)dt = 0$$

Therefore, if we wish to calculate positive and negative work done, then it is critical to know the exact time when P_m changes sign.

Example:

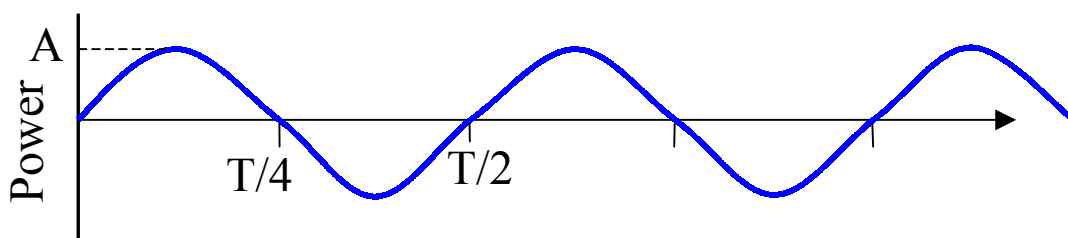
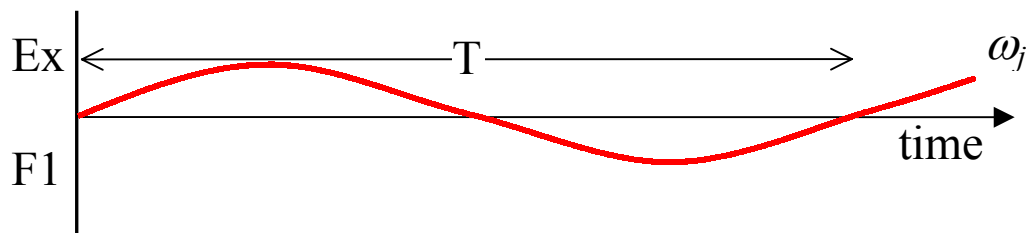
The power output of the muscles of a human movement involving flexion and extension about a joint can be approximated by a sine curve, i.e.

$$P_m = A \sin(4\pi t / T) \text{ W}$$

Where,

A = the peak power produced

T = the period of the movement.



Note the period of the power = $\frac{1}{2}$ period of the movement

(a) What is the work done over half a movement period?

$$\begin{aligned}
W_m &= \int_{t_i}^{t_f} P_m(t) dt \\
\rightarrow W_m &= \int_0^{T/2} A \sin(4\pi t/T) dt \\
&= A \int_0^{T/2} \sin(4\pi t/T) dt \\
&= A \left[-\cos\left(\frac{4\pi t}{T}\right) \times \frac{T}{4\pi} \right]_0^{T/2} \\
&= -A \frac{T}{4\pi} \left[\cos\left(\frac{4\pi t}{T}\right) \right]_0^{T/2} \\
&= -A \frac{T}{4\pi} \left[\cos\left(\frac{4\pi T}{T \times 2}\right) - \cos(0) \right] \\
&= -\frac{AT}{4\pi} (1 - 1) \\
&= 0
\end{aligned}$$

(b) What is the work done over the first $\frac{1}{4}$ of T .

$$\begin{aligned}
W_m &= \int_0^{T/4} A \sin\left(\frac{4\pi t}{T}\right) dt \\
&= -A \frac{T}{4\pi} \left[\cos\left(\frac{4\pi t}{T}\right) \right]_0^{T/4} \\
&= -\frac{AT}{4\pi} \left[\cos\left(\frac{4\pi T}{4 \times T}\right) - \cos(0) \right] \\
&= -\frac{4T}{4\pi} (-1 - 1) = \frac{AT}{2\pi} \quad (\text{+ve work})
\end{aligned}$$

(c) What is the work done over the second $\frac{1}{4}$ of T ?

$$\begin{aligned}
W_m &= -\frac{AT}{4\pi} \left[\cos\left(\frac{4\pi t}{T}\right) \right]_{T/4}^{T/2} \\
&= -\frac{AT}{4\pi} \left[\cos\left(\frac{4\pi T}{2T}\right) - \cos\left(\frac{4\pi T}{4T}\right) \right] \\
&= -\frac{AT}{4\pi} (1 - (-1)) = -\frac{AT}{2\pi} \quad (- \text{ ve work})
\end{aligned}$$

Explain why the work done over $\frac{1}{2}$ a period is zero.

The positive work of concentric contraction for the first $\frac{1}{4}$ of T is cancelled by the negative work of eccentric contraction for the second $\frac{1}{4}$ of T .

3.1.5 Mechanical work done on an external load.

When a body, or a segment of the body exerts a force on an external body, it can only do work if there is movement.

In this case the work is given by the product of the force acting and the displacement of the body in the direction of the applied force.

When a force \mathbf{F} acts over an infinitesimal displacement $d\mathbf{s}$ it does work

$$dW = \mathbf{F} \cdot d\mathbf{s}$$

If the force \mathbf{F} acts over a finite distance between points \mathbf{s}_1 and \mathbf{s}_2 on a line, then

$$W = \int_{s_1}^{s_2} \mathbf{F} \cdot d\mathbf{s}$$

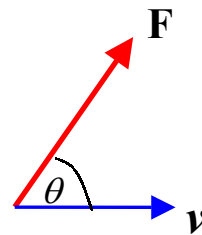
If the force \mathbf{F} which acts over an infinitesimal displacement $d\mathbf{s}$, does so in time dt , then the power is,

$$P = \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{s}}{dt} = \mathbf{F} \cdot \mathbf{v} \quad \text{In general both } \mathbf{F} \text{ and } \mathbf{v} \text{ are time dependent.}$$

i.e.

$$P = Fv\cos(\theta)$$

$$P = F_x v_x + F_y v_y \quad (\text{for 2D motion})$$



Then

$$\begin{aligned} W &= \int_{t_i}^{t_f} P dt = \int \mathbf{F} \cdot \mathbf{v} dt \\ &= \int_{t_i}^{t_f} Fv dt; \text{ if } \mathbf{F} \parallel \mathbf{v} \\ &= \int_{t_i}^{t_f} (F_x v_x + F_y v_y) dt; \text{ for general 2D motion} \end{aligned}$$

Example:

A ball is thrown with a constant accelerating force of $100N$ for a period of $180ms$. The mass of the ball is $1.0Kg$ and it starts from rest.

Calculate the work done on the ball during the time of force application.

Solution:

$$\mathbf{F} = m\mathbf{a} \Rightarrow \mathbf{a} = \frac{\mathbf{F}}{m} = \frac{100}{1.0} = 100 \text{ m/s}^2$$

$$\mathbf{s} = ut + \frac{1}{2}at^2$$

starts from rest $\Rightarrow u = 0 \text{ m/s}$

given $t = 0.18 \text{ s}$

$$\Rightarrow S_f = 0 + \frac{1}{2} \times 100 \times (0.18)^2 = 1.62 \text{ m}$$

Where S_f = the value of s when the ball leaves the hand.

$$\begin{aligned} \therefore W &= \int_0^s F ds; \text{ (assuming } \mathbf{F} \parallel d\mathbf{s} \text{)} \\ &= F \int_0^{S_f} ds \\ &= F [s]_0^{S_f} = FS_f = 100 \times 1.62 = 162 \text{ J} \end{aligned}$$

Example:

A ball of mass 1Kg is thrown with a force that varies time, as indicated below.

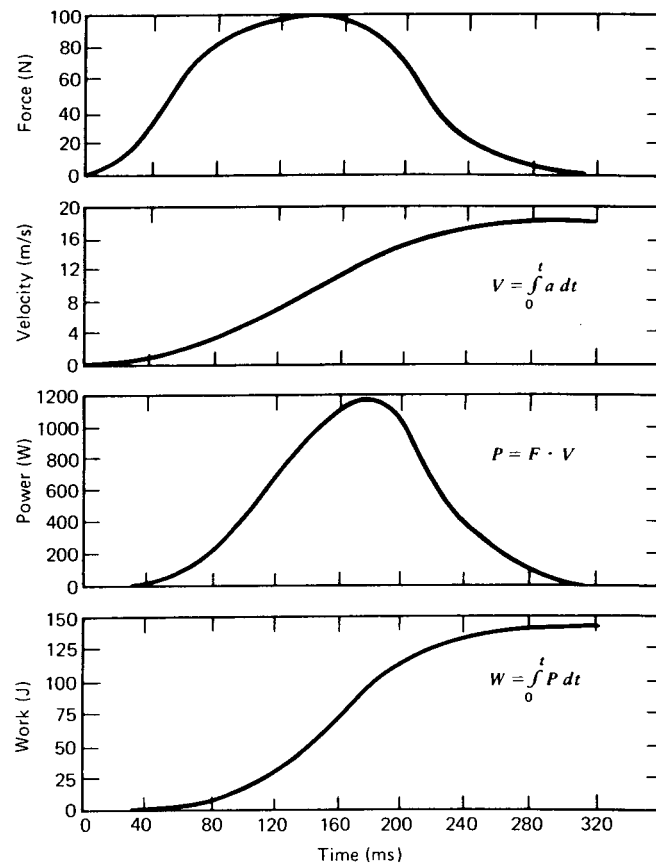


Figure 5.7 Forces, velocity, mechanical power, and work done on a baseball while being thrown. See text for details.

The velocity of the ball in the direction of the force is also plotted on the same time base and was calculated from

$$v(t) = \int_0^t a(t) dt = \int_0^t \frac{F(t)}{m} dt$$

$m = 1\text{Kg} \Rightarrow \mathbf{F}$ and \mathbf{a} have same numerical value.

Calculate the instantaneous power given to the ball and the total work done on the ball during the throwing period.

Solution:

$$P = Fv$$

For a given time t multiply the corresponding values of $F(t)$ and $v(t)$ to give $P(t)$:

$$P(t) = F(t)v(t)$$

$$W(t) = \int_0^t P(t) dt$$

At any time t , $W(t) =$ area under the $P(t)$ curve from $0 \rightarrow t$

What is the situation when the ball is caught?

Force is in the same direction (as above) – but velocity is in the opposite direction

\Rightarrow Power and work done are negative.

\Rightarrow ball is doing work on the body.

3.1.6 Mechanical energy transfer between segments.

Each body segment exerts forces on its neighboring segments. If there is a translational movement of the joint as a result of this then the mechanical energy transferred between segments i.e. one segment does work on another. This work is in addition to the muscular work described above.

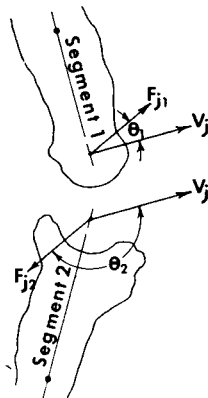


Figure 5.8 Reaction forces and velocities at a joint center during dynamic activity. The dot product of the force and velocity vectors is the mechanical power (rate of mechanical energy transfer) across the joint.

\mathbf{F}_{j1} = Reaction force at the joint of segment two on segment one.

\mathbf{F}_{j2} = Reaction force at the joint of segment one on segment two.

\mathbf{v}_j = velocity of joint.

θ_1 = angle between \mathbf{F}_{j1} and \mathbf{v}_j .

θ_2 = angle between \mathbf{F}_{j2} and \mathbf{v}_j .

Note: $\theta_1 + \theta_2 = 180^\circ$ because $\mathbf{F}_{j1} = -\mathbf{F}_{j2}$

Power going to segment 1:

$$P_1 = \mathbf{F}_{j1} \cdot \mathbf{v}_j = F_{j1} v_j \cos \theta_1$$

Power going to segment 2:

$$P_2 = \mathbf{F}_{j2} \cdot \mathbf{v}_j = F_{j2} v_j \cos \theta_2 = -F_{j1} v_j \cos \theta_1$$

That is

$$P_2 = -P_1$$

\Rightarrow Power flowing into one segment = power flowing out of the other.

\Rightarrow Net power production (by the process) = zero.

In an n joint system, the algebraic sum of the n power flows will be zero \Rightarrow these power flows are passive and do not add to or subtract from total body energy.

Thus joint reaction forces may transfer energy between segments, but they do not add energy to or take it from the body.

This mechanism of energy transfer between adjacent segments is quite important in the conservation of energy of any movement because it is passive and does not require muscle activity.

Example:

In walking, at the end of swing, the swinging foot and leg lose much of their energy by transfer upward through the thigh to the trunk, here it is conserved and converted to kinetic energy to accelerate the upper body forward.

3.2 Causes of inefficient movement

- Difficult to focus directly on efficiency.
- More expedient to focus on the causes of inefficiency and thereby improve the efficiency of the movement.

There are four major causes of mechanical inefficiency

1. Co-contractions

- Inefficient, because muscles fight against each other.

Suppose a certain movement can be accomplished with

$$\tau_f = 30 \text{ N.m (flexor torque)}$$

The most efficient way to do this is to have,

$$\tau_e = 0 \text{ N.m (extensor torque)}$$

$$\tau_f = 30 \text{ N.m (flexor torque)}$$

However any number of possibilities exist, e.g.

$$\tau_e = 10 \text{ N.m}$$

$$\tau_f = 40 \text{ N.m}$$

or

$$\tau_e = 20 \text{ N.m}$$

$$\tau_f = 50 \text{ N.m}$$

but these are inefficient.

In the latter case there is an unnecessary 20 N.m of τ in both the extensors and flexors.

⇒ The flexors are doing unnecessary positive work to overcome the negative work of the extensors.

Co-contractions occur in many pathologies ,e.g., hemiplegia & spastic cerebral palsy.

They also occur to some extent during normal movement, when it is necessary to stabilize a joint – especially in heavy lifting & explosive events.

At present measurement of co-contraction is only possible by monitoring the EMG activity of the antagonistic muscle – but this does not give quantitative info.

Winter and Falconer quantify co-contraction by,

$$\% \text{ contraction} = 2 \times \frac{\tau_{\text{antag}}}{\tau_{\text{agon}} + \tau_{\text{antag}}} \times 100\%$$

Example:

If

$$\tau_{\text{agon}} = 50 N.m$$

$$\tau_{\text{antag}} = 20 N.m$$

then

$$\% \text{ contraction} = 2 \times \frac{20}{50 + 20} \times 100\% = \frac{40}{70} \times 100\% = 57\%$$

2. Isometric Contractions against Gravity

In normal movement minimal muscle activity is devoted to holding limb segments against gravity – momentum of body & limbs allows for a smooth interchange of energy.

- Unnecessary isometric tension of muscles is inefficient!

WHY ?

- In many pathologies movements are so slow that there are periods when the limbs and/or the trunk are held in near-isometric contraction.

Example:

People with spastic cerebral palsy often over crouch with knees flexed

⇒ Excessive quadriceps activity is required to stop them from falling.

In isometric contraction against g no work is done! (no movement). EMG shows “extra” muscle activity, therefore “extra” metabolic energy is being consumed.

No valid technique exist to separate the metabolic cost of this inefficiency.

3.Generation of Energy at One Joint and Absorption at Another

This type of inefficiency arises when positive work at one joint occurs at the same time as negative work at others.

– It’s an extension of co-contraction.

Example:

During normal walking it occurs during double support, when the energy increase of push-off at one leg (positive W) occurs at the same time as energy absorption (negative W) at heel strike of the other.

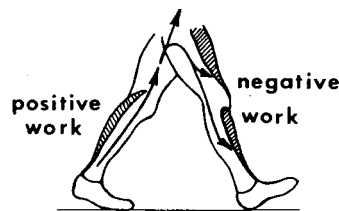
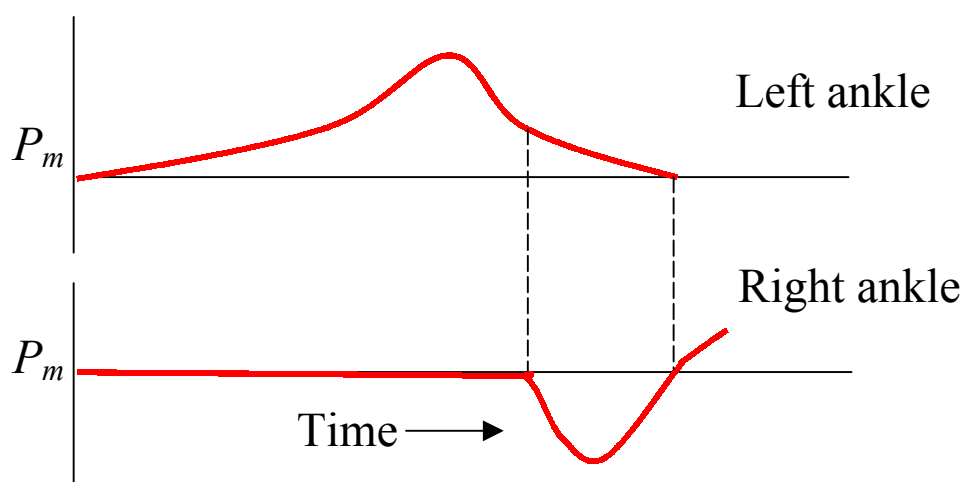


Figure 5.11 Example of a point in time during gait that positive work by the push-off muscles can be canceled by negative work of the weight-accepting muscles of the contralateral leg. (Reproduced by permission of *Physiotherapy Canada*.)

In the above figure:

Positive work being mainly done by the left plantar flexors is being canceled to some extent by negative work of the right dorsiflexors and knee extensors.

(Positive work about left ankle, partially cancelled by negative work about the right knee and right ankle)



The only way to analyse such inefficiencies is to calculate P_m at each joint separately & to quantify the overlap of simultaneous phases of positive and negative W .

Keep in mind that such inefficiencies are often necessary for stability and safety – activities like walking & running are complex, requiring several functions to be done simultaneously.

4. Jerky Movements

Efficient energy exchanges are characterized by smooth-looking movements (e.g. ballet dancer, high jumper, weight lifting). Energy added to the body by positive W at one point in time is conserved and little is lost by muscles doing negative W .

On the other hand, jerky movements are inefficient – energy added at one time is removed a fraction of a second later.

Succession of starts & stops

⇒ bursts of +ve and -ve W

↘ excess metabolic cost.

This energy cost can be assessed by:

1. Segment-by-segment energy analysis
2. Joint-by-joint power analysis.

Done later ?

3.3 Forms of energy storage.

1. Potential energy – due to gravity.

$$PE = mgh$$

h = height above some reference point.

The $h = 0$ reference point should be carefully chosen to fit the problem in question.

Normally it is taken as the lowest point the body takes during the given movement.

2. Kinetic energy – energy of movement.

Two forms:

1) Translational Kinetic energy = $\frac{1}{2}mv^2$

2) Rotational kinetic energy = $\frac{1}{2}I\omega^2$

Note:

1. Kinetic energy increases as the velocity squared
2. Polarity of direction of velocity is unimportant as velocity squared is always positive.
3. Lowest level of kinetic energy = 0, when the body is at rest.

1. Total energy and exchange within a segment.

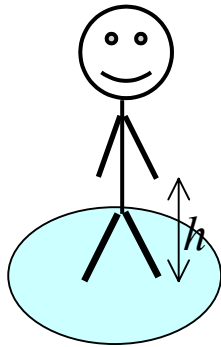
The total energy of a body or segment is,

$$E_s = PE + TKE + RKE$$

$$= mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

It is possible for a body to exchange energy within itself and still maintain a constant E_s ?

Consider a person spinning on an angular momentum table.



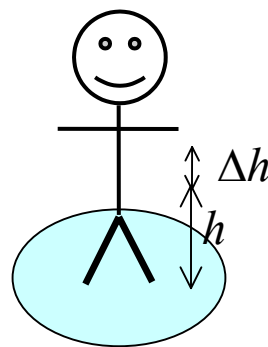
$$I = I_0$$

$$\omega = \omega_0$$

$$L = I_0\omega_0$$

$$RKE = \frac{1}{2}I_0\omega_0^2$$

$$= RKE_0$$



$$I = 2I_0$$

$$\omega = \frac{1}{2}\omega_0$$

$$L = I\omega = I_0\omega_0$$

$$RKE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 2I_0 \left(\frac{1}{2}\omega_0 \right)^2$$

$$= \frac{1}{4}I_0\omega_0^2 = \frac{1}{2}RKE_0$$

\Rightarrow half of RKE_0 is gone! Where?

$PE = mgh =$ potential energy before abduction

$PE = mg(h + \Delta h) =$ potential energy after abduction

conservation of energy ($PE + RKE = const.$) \Rightarrow

$$mg\Delta h = \frac{1}{2} RKE_0$$

Example:

A spherical ball of mass 300g and radius 5cm is thrown into the air. It reaches a height of 20m, at which it has a forward speed of 20m/s and a backspin of 5rev/s. What is the total energy of the ball?

Solution:

$$E_s = PE + TKE + RKE$$

Taking the ground to be $h = 0$, we have

$$PE = mgh = 0.3 \times 9.8 \times 20 = 58.8J$$

$$TKE = \frac{1}{2}mv^2 = \frac{0.3}{2} \times (20)^2 = 60.0J$$

$$RKE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{2}{5}mr^2 \times (5 \times 2\pi)^2$$

$$= \frac{1}{5} \times 0.3 \times (0.05)^2 \times (10\pi)^2 = 0.15J$$

$$\rightarrow E_s = 58.8 + 60.0 + 0.15 = 118.95J$$

(assume the sphere to be a uniform solid)

Note: For this ball traveling up and down there will be a transfer of energy between PE and TKE .

How can there be a change in *RKE*?

3.3.1 Energy of a body segment and exchanges of energy within the segment.

In most kinds of human movement, most body segments contain all three energies in various combinations at any given time.

Throughout such movements there is usually exchange of energy between the three types.

For multi-segment systems such exchanges can be very complex – there can be exchanges within a segment or between adjacent segments.

Example:

During gait, *HAT* has 2 peaks of *PE* each stride – during mid-stance of each leg. At this time *HAT* has slowed its v_x to a minimum. Then as the body falls forward to double support *HAT* increases v_x at the expense of *h*.

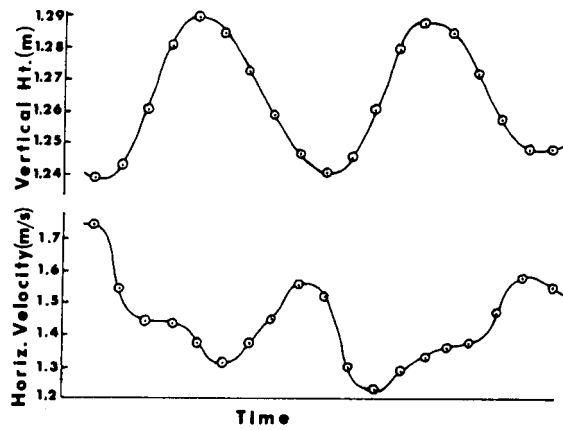


Figure 5.13 Plot of vertical displacement and horizontal velocity of HAT shows evidence of energy exchange within the upper part of the body during gait.

From the fig:

$PE \sim \text{sinusoidal}$
 $\left\{ \begin{array}{l} \text{Peaks at mid-stance} \\ \text{Minima at double-stance} \end{array} \right.$

$v_x \sim \text{completely out of phase } (180^\circ)$
 $\left\{ \begin{array}{l} \text{Peaks at} \\ \text{double-stance} \\ \text{minima at} \\ \text{mid-stance} \end{array} \right.$

- Exchanges of energy within a segment are characterized by opposite changes of PE and KE components.
- If the exchange is perfect (as in a swinging frictionless pendulum) then E_s is constant \Rightarrow no energy is being added or lost.

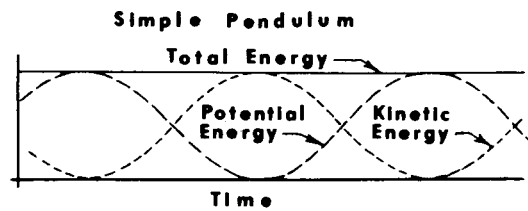


Figure 5.14 Exchange of kinetic and potential energies in a swinging frictionless pendulum. Total energy of the system is constant, indicating that no energy is being added or lost. (Reproduced by permission of *Physiotherapy Canada*.)

- Consider the other extreme – in which no energy exchange takes place. This situation is characterized by ‘totally’ in-phase energy components – not necessarily of equal magnitude.

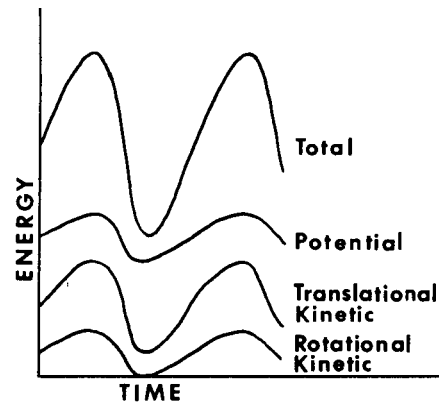


Figure 5.15 Energy patterns for a segment in which no exchanges are taking place. All energy components are perfectly in phase.

3.3.1.1 Estimation of energy exchange.

Within a segment the energy exchange can be estimated if the peak-to-peak change in each energy component is known (over some Δt).

Peak-to-peak

$$E_{ex} = \Delta E_p + \Delta E_t + \Delta E_r - \Delta E_s$$

Energy exchange

If there is no exchange ($E_{ex} = 0$), then

$$\Delta E_p + \Delta E_t + \Delta E_r = \Delta E_s$$

If there is 100% exchange, then $\Delta E_s = 0$.

Example:

A visual scan of the energies of the leg segment during walking yields the following maximum and minimum energies on the stride period.

$E_s (\text{max}) = 29.30J$	$E_s (\text{min}) = 13.14J$
$E_p (\text{max}) = 15.18J$	$E_p (\text{min}) = 13.02J$
$E_t (\text{max}) = 13.63J$	$E_t (\text{min}) = 0.09J$
$E_r (\text{max}) = 0.95J$	$E_r (\text{min}) = 0J$

Estimate the energy exchange.

Solution:

We have,

$$\Delta E_s = 29.30 - 13.14 = 16.16 \text{ J}$$

$$\Delta E_p = 15.18 - 13.02 = 2.16 \text{ J}$$

$$\Delta E_t = 13.63 - 0.09 = 13.54 \text{ J}$$

$$\Delta E_r = 0.95 - 0 = 0.95 \text{ J}$$

\Rightarrow

$$\Delta E_p + \Delta E_t + \Delta E_r = 2.16 + 13.54 + 0.95 = 16.65 \text{ J}$$

So

$$\begin{aligned} E_{ex} &= 16.65 - 16.16 \\ &= 0.49 \text{ J} \end{aligned}$$

i.e. 0.49 J of energy was exchanged during the stride!

or

$$\frac{0.49}{16.65} \times 100 = 2.9\% \text{ was exchanged.}$$

\Rightarrow Highly non-conservative!

3.3.1.2 Exact formula for energy exchange within segments.

The above example illustrates a simple situation – only one minimum and maximum over time period (Δt).

If energy components have several maximum and minimum – must calculate the sum of absolute energy changes over Δt .

The work W_s done on and by a segment during N sample periods is,

$$W_s = \sum_{i=1}^N |\Delta E_s|$$

If no energy exchanges occur (between components) the work done on and by the segment is

$$W_s' = \sum_{i=1}^N (|\Delta E_p| + |\Delta E_t| + |\Delta E_r|)$$

⇒ The energy conserved is,

$$W_c = W_s' - W_s$$

⇒ The percentage conserved is,

$$C_s = \frac{W_c}{W_s'} \times 100\%$$

If $W_s' = W_s \Rightarrow$ all three components are in phase and no energy is conserved.

Conversely, if $W_s' = 0$, $W_c = W_s' \Rightarrow 100\%$ of energy is being conserved.

3.3.2 Total energy of a multi-segment system.

If E_{si} = total energy of the i^{th} segment, then the total body energy E_b at t is,

$$E_b = \sum_{i=1}^B E_{si}$$

Where

B = number of body segments.

Note: Individual segments continuously change their energy with time $\Rightarrow E_b$ changes with time.

However in interpreting the changes in E_b care must be taken because:

- 1) There is considerable potential for energy transfer between segments → efficiency in movement.
- 2) There are a number of possible generators and absorbers of energy at each joint → inefficiency in movement.

The transfer of energy between segments will not change E_b . There may be several simultaneously concentric and eccentric contractions, e.g. in a given time period two muscles groups may generate 30 J while a third may absorb 20 J ⇒ net change of 10 J.

Only a detailed analysis of mechanical power at the joints will tell us the extent of such cancellation.

Consider a simple muscular system represented by a pendulum mass and a pair of antagonistic muscle groups m_1 and m_2 , crossing a simple hinge joint.

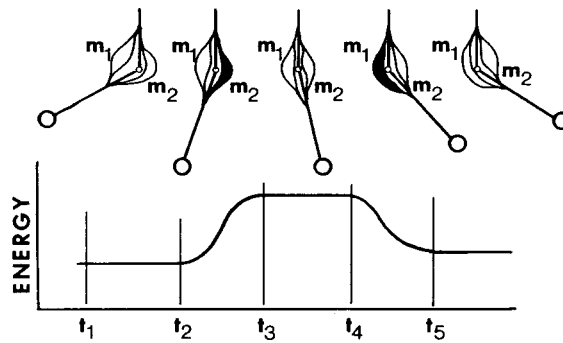


Figure 5.16 Pendulum system with muscles. When positive work is done, the total energy increases; when negative work is done, the total energy decreases.

The figure shows this arrangement along with the time history of the total energy of the system.

At t_1 the segment is rotating anticlockwise at some ω_s – no muscle activity until $t_2 \Rightarrow$ normal pendulum exchange between t_1 and t_2 .

Therefore constant E_b .

At t_2 , m_2 contracts concentrically increasing both KE and PE , this continues until t_3 . τ_m and ω_s have the same sense of rotation \Rightarrow +ve work between t_2 and t_3 .

$\therefore E_b$ increases.

At t_3 , m_2 stops contracting and there is no muscle activity until $t_4 \Rightarrow$ pendulum exchange.

Therefore E_b is constant between t_3 and t_4 (at higher level).

At t_4 , m_1 contracts eccentrically to slow segment \Rightarrow loss of KE . τ_m and ω_s have opposite senses of rotation therefore negative work is done between t_4 and t_5 .

$\therefore E_b$ decreases.

After t_5 muscles relax and pendulum exchange occurs

\Rightarrow constant E_b until contraction.

This simple model can be extended to more complex systems.

In general:

- 1) Positive work done by muscles increases E_b .
- 2) Negative work done by muscles decreases E_b .
- 3) For cyclical activity (e.g. level running) ΔE_b per cycle (stride) = 0 \Rightarrow +ve W = -ve W.

3.5 Power Balance at Joint and within Segments.

So far we have considered (with respect to energy and power):

- 1) Conservation of mechanical energy within a segment (section 3.0.2)
- 2) Muscle mechanical power (section 3.1.3)
- 3) Passive energy transfer across joints (section 3.1.6)

Here we look at the transfer of energy from segment to segment due to active muscles – in addition to their role of generation and absorption of energy.

3.5.1 Energy Transfer via muscles.

Muscles can transfer energy from one segment to another if the two segments are rotating in the same direction.

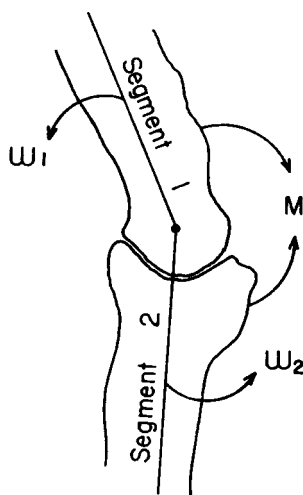


Figure 5.18 Energy transfer between segments occurs when both segments are rotating in the same direction and when there is a net moment of force acting across the joint. See text for detailed discussion.

The figure shows two segments rotating in the same direction but with different ω 's.

The product of $\tau_m \omega_2$ is positive (τ_m and ω_2 have same polarity)

\Rightarrow Energy is flowing into segment 2 from muscles producing τ_m .

On the other hand, $\tau_m \omega_1$ is negative (τ_m and ω_1 have opposite polarity).

\Rightarrow Energy is flowing out of segment 1 and entering the muscle.

If $\omega_1 = \omega_2$ (isometric contraction)

\Rightarrow Energy rate into 2 = energy rate out of 1

If $\omega_1 > \omega_2$ (muscles lengthening)

\Rightarrow Absorption and transfer take place

or energy rate into 2 = energy rate out of 1

– absorption at muscle

If $\omega_1 < \omega_2$ (muscles shortening)

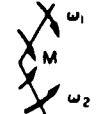
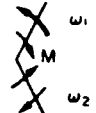
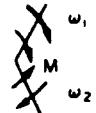
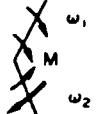
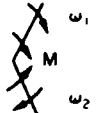

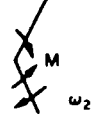

\Rightarrow Generation and transfer take place.

or energy rate into 2 = energy rate out of 1

+ generation at muscle.

The following table gives a summary of all possible power functions that can occur at a joint.

TABLE 5.2 Power Generation, Transfer and Absorption Functions

Description of Movement	Type of Contraction	Directions of Segmental Angular Velocities	Muscle Function	Amount, Type, and Direction of Power
Both segments rotating in opposite directions (a) joint angle decreasing	Concentric		Mechanical energy generation	$M\omega_1$ generated to segment 1 $M\omega_2$ generated to segment 2
(b) joint angle increasing	Eccentric		Mechanical energy absorption	$M\omega_1$ absorbed from segment 1 $M\omega_2$ absorbed from segment 2
Both segments rotating in same direction (a) joint angle decreasing (e.g. $\omega_1 > \omega_2$)	Concentric		Mechanical energy generation and transfer	$M(\omega_1 - \omega_2)$ generated to segment 1 $M\omega_2$ transferred to segment 1 from 2
(b) joint angle increasing (e.g. $\omega_2 > \omega_1$)	Eccentric		Mechanical energy absorption and transfer	$M(\omega_2 - \omega_1)$ absorbed from segment 2 $M\omega_1$ transferred to segment 1 from 2
(c) joint angle constant ($\omega_1 = \omega_2$)	Isometric (dynamic)		Mechanical energy transfer	$M\omega_2$ transferred from segment 2 to 1.
One segment fixed (e.g. segment 1.) (a) joint angle decreasing ($\omega_1 = 0, \omega_2 > 0$)	Concentric		Mechanical energy generation	$M\omega_2$ generated to segment 2.
(b) joint angle increasing ($\omega_1 = 0, \omega_2 < 0$)	Eccentric		Mechanical energy absorption	$M\omega_2$ absorbed from segment 2.
(c) joint angle constant ($\omega_1 = \omega_2 = 0$)	Isometric (static)		No mechanical energy function	Zero.

From Robertson and Winter (1980). (Reproduced by permission from *J. Biomechanics*.)

In section 3.1.3 ‘muscle mechanical power’ we had the equation for the muscle power at a joint,

$$P_m = \tau_j \omega_j$$

In view of the above, we modify this equation to include the angular velocities of the adjacent segments,

$$P_m = \tau_j (\omega_1 - \omega_2)_j$$

Thus if ω_1 and ω_2 have the same polarity, the rate of transfer will be the lesser of the two power components.

3.5.2 Power Balance within Segments.

Energy can enter or leave a segment at muscles and across joints at the proximal end and distal ends.

Passive transfer across joints,

$$P = \mathbf{F} \cdot \mathbf{v}$$

Where \mathbf{F} = joint reaction force

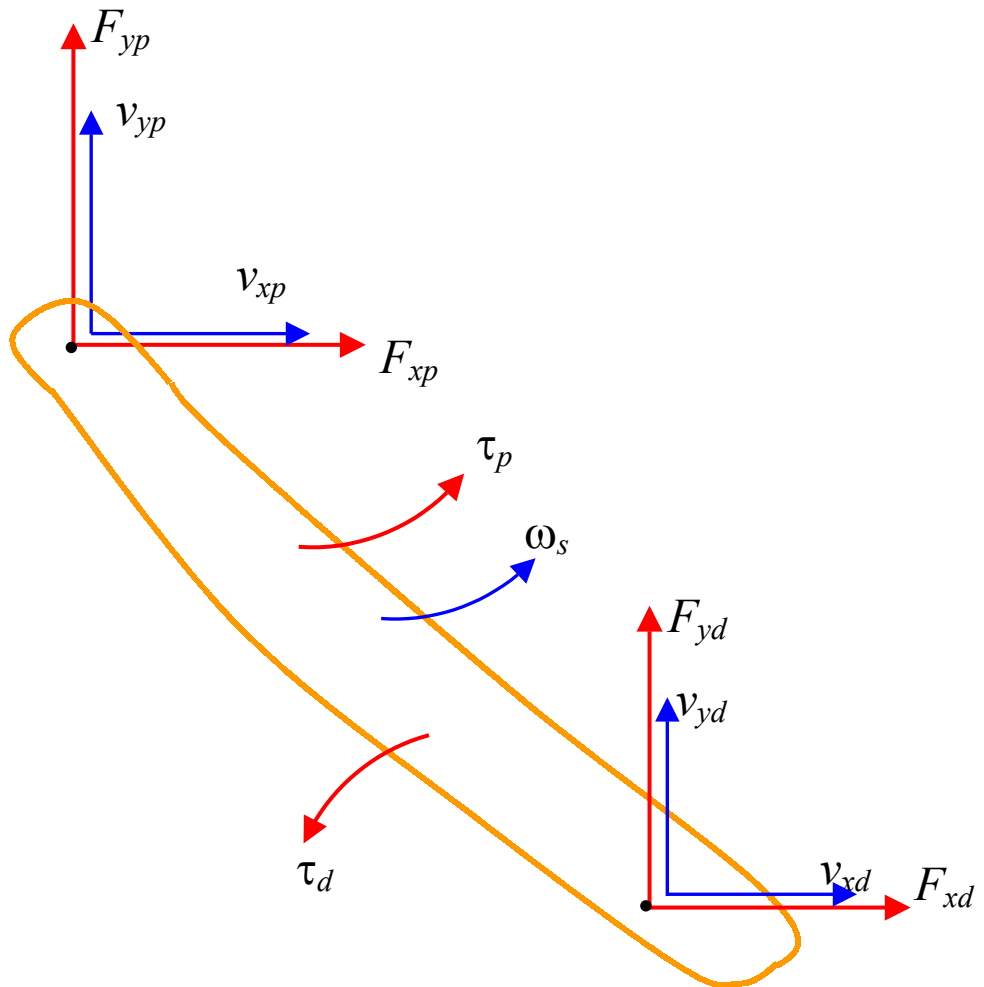
\mathbf{v} = velocity of joint centre of
rotation.

and active transfer plus absorption or generation,

$$P_m = \tau_j (\omega_1 - \omega_2)$$

must be calculated.

Consider the following as the state of a given segment at some point and time.



$$P_p = \mathbf{F}_p \cdot \mathbf{v}_p = F_{xp} v_{xp} + F_{yp} v_{yp}$$

$$P_{m_p} = \tau_p \omega_s$$

$$P_d = \mathbf{F}_d \cdot \mathbf{v}_d = F_{xd} v_{xd} + F_{yd} v_{yd}$$

$$P_{m_d} = \tau_d \omega_s$$

Reaction forces and velocities at joint centers (proximal and distal) are shown, plus muscle torques (proximal and distal) and segment ω , are shown.

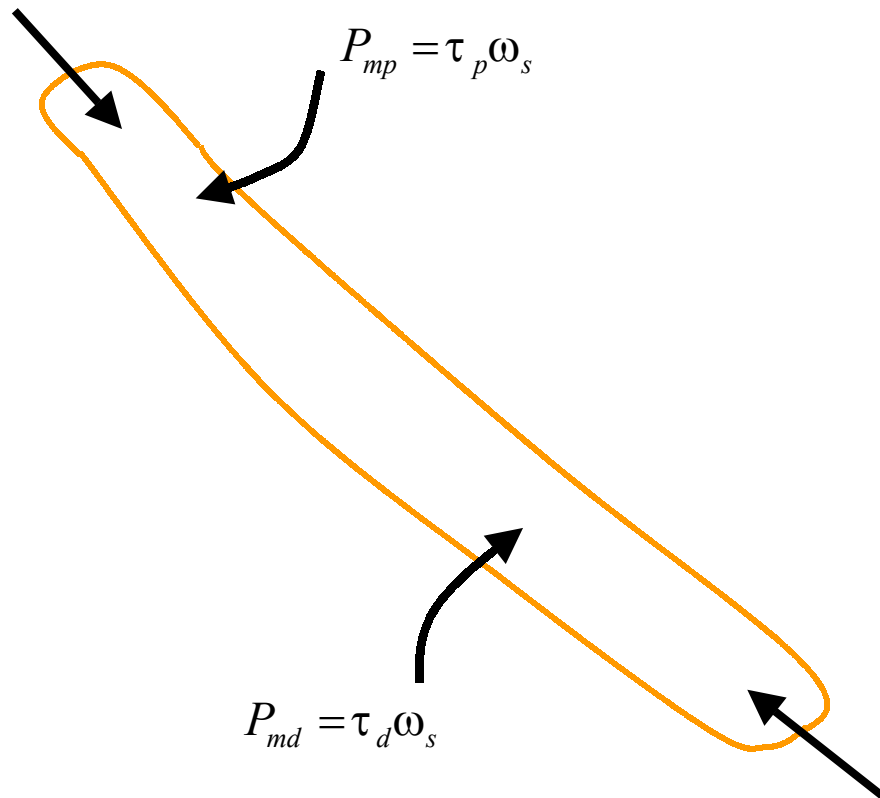
The total energy,

$$E_s = mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

needs to be known.

The power balance of the segment is,

$$P_{jp} = F_{xp}v_{xp} + F_{yp}v_{yp}$$



$$P_{jd} = F_{xd}v_{xd} + F_{yd}v_{yd}$$

The arrows show the direction of the positive power flows (energy entering the segment at joints and tendons).

If the force-velocity or the torque- ω products turn out to be negative \Rightarrow energy flow is leaving the segment.

Conservation of energy \Rightarrow

$$\frac{dE_s}{dt} = P_{jp} + P_{mp} + P_{jd} + P_{md}$$

Example:

Carry out a power balance for the leg and the thigh segments for frame 5, i.e. deduce the dynamics of energy flow for each segment separately and determine the power dynamics of the knee muscles (generation, absorption or transfer?)

Solution:

The information that we require is:

- 1) Joint centre velocities (hip, knee and ankle)
- 2) Reaction forces at joints
- 3) Muscle torques on segments
- 4) Segment angular velocity

Frame 5

Table A.2a, hip velocities:

$$v_{xh} = 1.36 \text{ m/s}, \quad v_{yh} = 0.27 \text{ m/s}$$

Table A.2b, knee velocities:

$$v_{xk} = 2.61 \text{ m/s}, \quad v_{yk} = 0.37 \text{ m/s}$$

Table A.2c, ankle velocities:

$$v_{xa} = 3.02 \text{ m/s}, \quad v_{ya} = 0.07 \text{ m/s}$$

Table A.3b, leg angular velocity:

$$\omega_l = 1.24 \text{ rad/s}$$

Table A.3c, thigh angular velocity:

$$\omega_t = 3.98 \text{ rad/s}$$

Table A.5a, leg segment reaction forces and torques:

$$\begin{array}{ll} F_{xk} = 15.1 \text{ N} & F_{yk} = 14.6 \text{ N} \\ F_{xa} = -12.3 \text{ N} & F_{ya} = 5.5 \text{ N} \\ \tau_a = -1.1 \text{ N.m} & \tau_k = 5.8 \text{ N.m} \end{array}$$

Table A.5b, thigh segment reaction forces and torques:

$$\begin{array}{ll} F_{xh} = -9.4 \text{ N} & F_{yh} = 102.8 \text{ N} \\ F_{xk} = -15.1 \text{ N} & F_{yk} = -14.6 \text{ N} \\ \tau_h = 8.5 \text{ N.m} & \tau_k = -5.8 \text{ N.m} \end{array}$$

For each segment the power flow is,

$$P_F = P_{jp} + P_{mp} + P_{jd} + P_{md}$$

Thus, leg power flow is,

$$\begin{aligned}
P_F &= P_{jk} + P_{mk} + P_{ja} + P_{ma} \\
&= \mathbf{F}_k \cdot \mathbf{v}_k + \tau_k \omega_l + \mathbf{F}_a \cdot \mathbf{v}_a + \tau_a \omega_l \\
&= F_{xk} v_{xk} + F_{yk} v_{yk} + \tau_k \omega_l + F_{xa} v_{xa} + F_{ya} v_{ya} + \tau_a \omega_l \\
&= 15.1 \times 2.61 + 14.6 \times 0.37 + 5.8 \times 1.24 - 12.3 \times 3.02 \\
&\quad + 5.5 \times 0.07 - 1.1 \times 1.24 \\
&= 44.81 + 7.19 - 36.76 - 1.36 \\
&= 13.88 \text{ W}
\end{aligned}$$

Rate of change of total energy (E_l) for leg?

$$\frac{dE_l}{dt} = P_{jk} + P_{mk} + P_{ja} + P_{ma} = 13.88 \text{ W}$$

We can obtain dE_l/dt from table A.6

$$E_l(\text{frame 6}) = 20.5 \text{ J}, \quad E_l(\text{frame 4}) = 20.0 \text{ J}$$

So,

$$\frac{dE_l}{dt} \sim \frac{\Delta E_l(\text{frame 5})}{\Delta t} = \frac{E_l(\text{frame 6}) - E_l(\text{frame 4})}{\text{time of two frames}}$$

i.e.

$$\frac{\Delta E_l(\text{frame 5})}{\Delta t} = \frac{20.5 - 20.0}{0.0286} = 17.5 \text{ W}$$

$$\text{Balance} = 17.5 - 13.88 = 3.6 \text{ W}$$

Thigh power flow is,

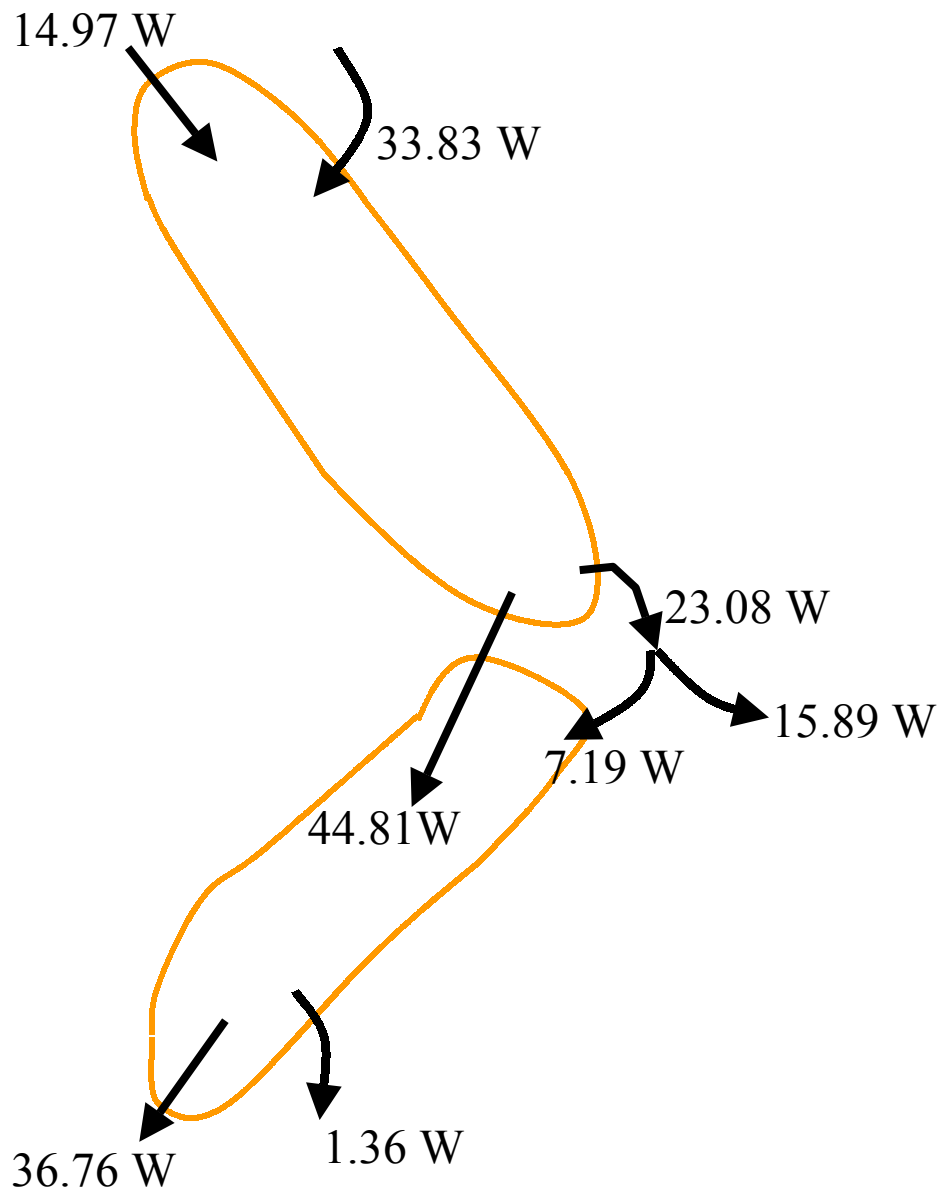
$$\begin{aligned}
P_F &= P_{jh} + P_{mh} + P_{jk} + P_{mk} \\
&= \mathbf{F}_h \cdot \mathbf{v}_h + \tau_h \omega_t + \mathbf{F}_k \cdot \mathbf{v}_k + \tau_k \omega_t \\
&= (-9.4, 102.8) \cdot (1.36, 0.27) + 8.5 \times 3.98 \\
&\quad + (-15.1, -14.6) \cdot (2.61, 0.37) - 5.8 \times 3.98 \\
&= 14.97 + 33.83 - 44.81 - 23.08 \\
&= -19.09 \text{ W}
\end{aligned}$$

From table A.6,

$$\begin{aligned}
E_t(\text{frame 6}) &= 47.4 \text{ J}, \quad E_t(\text{frame 4}) = 47.9 \text{ J} \\
\rightarrow \frac{dE_t(\text{frame 5})}{dt} &\sim \frac{\Delta E_t}{\Delta t} = \frac{E_t(\text{frame 6}) - E_t(\text{frame 4})}{0.0286} \\
&= \frac{47.4 - 47.9}{0.0286} = -17.5 \text{ W}
\end{aligned}$$

$$\text{Balance} = \frac{\Delta E_t}{\Delta t} - P_F = -17.5 - (-19.09) = 1.59 \text{ W}$$

Summary of power flows:



23.08 W leaves the thigh into the knee extensors
(τ_k (of leg) = +ve).

7.19 W enters the leg from the same extensors

\Rightarrow Knee extensors actively transfer 7.19 W and
simultaneously absorb 15.88 W.

4. Throw-like and Push-like Movement Patterns.

When a performer ‘throws’ or ‘pushes’ an object the objectives are:

1. To project the object the greatest vertical or horizontal distance (e.g. javelin, discus, shot-put)
or
2. To project an object primarily for accuracy – speed of projection may enhance the effectiveness of this (e.g. darts, cricket, baseball, basketball)

Within each of these groups of skills, biomechanical factors and principles govern how the body’s segmental movements best produce the desired accuracy or speed required.

Let’s look at some terminology:

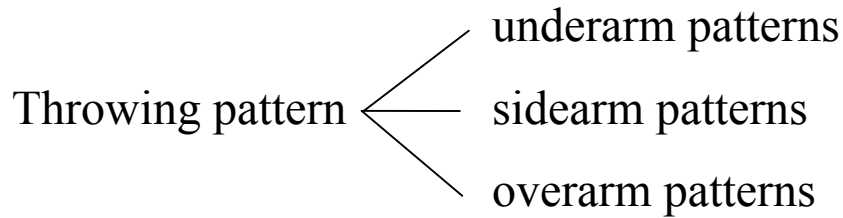
- **Movement pattern** – a general series of anatomical movements that have common elements of spatial configuration (e.g. same plane of motion).

Within a movement pattern, individual segmental movements may vary slightly in ROM’s, velocities and planes of motion.

Throwing, kicking and pushing are all general movement patterns. These patterns may be further subdivided,

according to where the movements occur relative to the body.

e.g.



- Skill – when a general movement pattern is adapted within the constraints of some particular movement activity or sport it is called a skill.

Sport Skills Classified Under Four General Movement Patterns

Underarm Patterns	Sidearm Patterns	Overarm Patterns	Kicking Patterns
Softball pitch	Baseball throw	Baseball pitch	Football punt
Bowling ball delivery	Discus throw	Shot put	Soccer placekick
Horseshoe pitch	Hammer throw	Javelin throw	Swimming flutter kick
Volleyball serve	Volleyball serve	Volleyball serve	Walking or running, swing phase
Volleyball bump	Baseball batting	Volleyball spike	Dolphin kick
Badminton underarm clear	Tennis drive	Basketball one-hand shot	
Badminton serve	Badminton drive	Tennis serve	
Field hockey drives and passes	Racquetball shots	Tennis smash	
	Squash shots	Badminton smash and around-the-head shot	
	Handball shots	Cricket "pitch"	

- Constraints – factors that influence the time and space variables e.g.
 - Mass of object
 - Mass of implement
 - Size of object
 - Size of target
 - Size of playing area

Open and closed chain movements:

- Open skills – performed in an unpredictable environment (e.g. catching, striking, basketball shot)
- Closed skills – performed in a predictable environment (e.g. archery, basketball free throw, hammering a nail)
- Open kinetic chain exercise – body segments move in combination and end segment is free to move in space
- Closed kinetic chain exercise – end segment meets with ‘considerable’ resistance, which prohibits or restrains its free motion.

Usually:

Throw-like patterns are open

Push-like patterns can be either (maybe emphasis on closed)

4.1 Throw-like patterns: Sequential Segmental Rotations.

- Throw-like pattern – object or end segment starts from behind proximal segment and ends up in front.
- Includes kicking, striking and batting.

Objectives:

Project an object for greatest distance (horizontal or vertical)

Project an object for accuracy – where velocity of object enhances its effectiveness

Velocity of object at release is most important variable.
Velocity of release depends upon velocity of contact point that object has with hand, foot, or implement being used
→ high ‘end-point’ velocity.

4.1.1 Open Kinetic Link Model.

To illustrate the generation of high end-point velocity, consider a three segment system, with segments A, B and C with axis of rotations a, b and c.

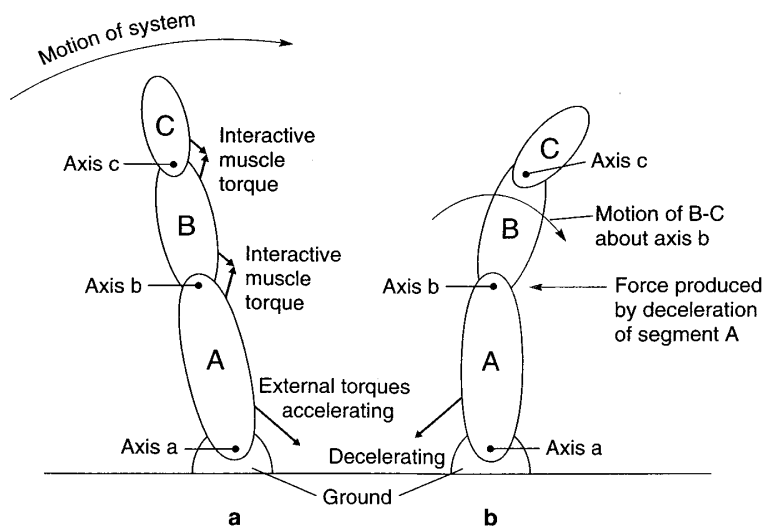


FIGURE J.1 A three-segment model of the kinetic link concept with external and internal torques influencing its motion.

Muscle torques between segment A and ‘ground’ are external to system and can therefore change L.

Muscle torques between A and B, and B and C are inter-segmental (internal to system) and therefore do not change \mathbf{L} .

To see how it functions consider the following situation:

1. Muscles to the right of segment A apply external τ large enough to accelerate segment A clockwise.
2. Inter-segmental muscle τ 's A–B and B–C work to resist lagging back or anticlockwise motion of distal segments relative to proximal segment as system is accelerated clockwise. If τ_m A–B and B–C were not applied segments B and C would lag back.
3. The external τ applied to A accelerates the entire system and gives it its \mathbf{L}_0 (\mathbf{L}_{A+B+C})
4. Now suppose a second external τ acts to left of axis a. This decelerates A and tends to 'fix' axis b in space. Segments B and C are free to rotate about axis b.
5. Segments B and C have smaller I about axis b than axis a.

Conservation of $\mathbf{L} \Rightarrow$ angular velocity of segments B and C increases.

\rightarrow High end-point velocity.

$$I_{B+C} = m_{B+C} k_{B+C}^2$$

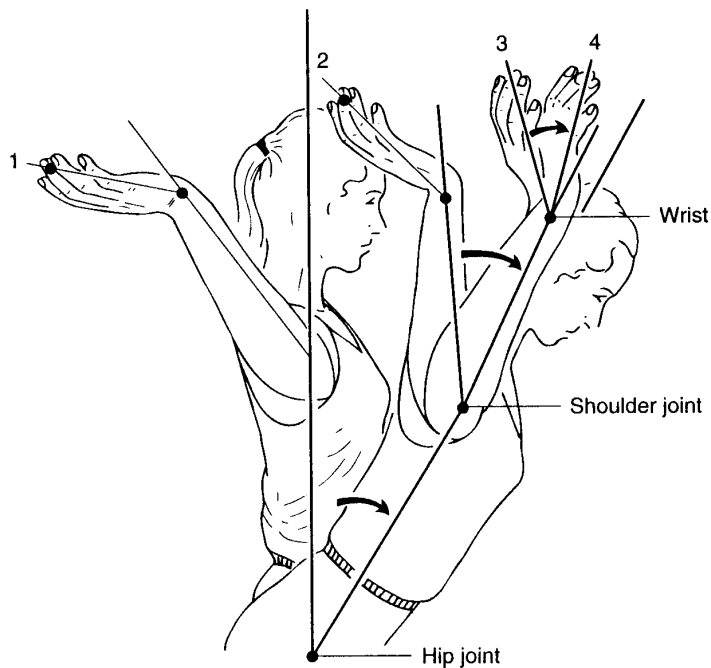
$$L_{B+C} = I_{B+C} \omega_{B+C} \rightarrow \text{axis of rotation at a}$$

$$L_{B+C} = I'_{B+C} \omega'_{B+C} \rightarrow \text{axis of rotation at b}$$

$$I'_{B+C} < I_{B+C} \therefore \omega'_{B+C} > \omega_{B+C}$$

Consider the above model applied to the following human model

Flexion of the hip, medial rotation of the shoulder joint, and flexion of the wrist with all distal segments of the upper extremity fixed.



A = trunk

a = hip joint

B = arm

b = shoulder joint

C = hand

c = wrist joint

Hip flexors produce the initial angular acceleration of system.

What is the end-point velocity?

It is linear! We have been talking about angular velocity.

Recall,

$$v = r\omega$$

Where:

r = radius of rotation

= perpendicular distance between contact point of object being projected and axis of rotation.

Implements such as bats and racquets give large end-point speeds because they have large r .

Suppose in the above figure we stabilize the chain and allow each segment to move one at a time, with each joint under going a 30° (0.524rad) rotation in 0.5sec .

$$\rightarrow \omega = \frac{0.524}{0.5} = 1.048 \text{ rad/s for each joint}$$

$$r_{hip} = 95 \text{ cm (}\perp\text{ distance from hip to fingers)}$$

$$r_{sh} = 48 \text{ cm (}\perp\text{ distance from shoulder to fingers)}$$

$$r_{wr} = 14 \text{ cm (}\perp\text{ distance from wrist to fingers)}$$

\rightarrow

$$v_{hip} = 95 \times 1.048 = 99.6 \text{ cm/s}$$

(velocity at fingers due to rotation about hip only)

$$v_{sh} = 48 \times 1.048 = 50.3 \text{ cm/s}$$

$$v_{wr} = 14 \times 1.048 = 14.67 \text{ cm/s}$$

Clearly large $r \rightarrow$ large v

So, to maximize end-point speed –maximize both the angular velocity and r . To achieve this timing (coordination between segments) is important.

4.2 Sequencing Segmental Rotations: The Kinetic Link Principle.

Open kinetic link systems have the following characteristics.

1. System links have a base, or fixed end and a free or open end.
2. More massive segments are proximal – less massive ones are distal.
3. External torque applied to base segment initiates the systems motion.
4. Distal segments lag behind (initially)

If the distal joints are free to rotate, the kinetic links act like a whip.

- Tapering of distal masses \Rightarrow smaller I as segment becomes more distal \rightarrow increasing ω of distal segment.
- As proximal segments decelerate, axis of rotation moves distal \rightarrow decreasing I and increasing ω of distal segment.

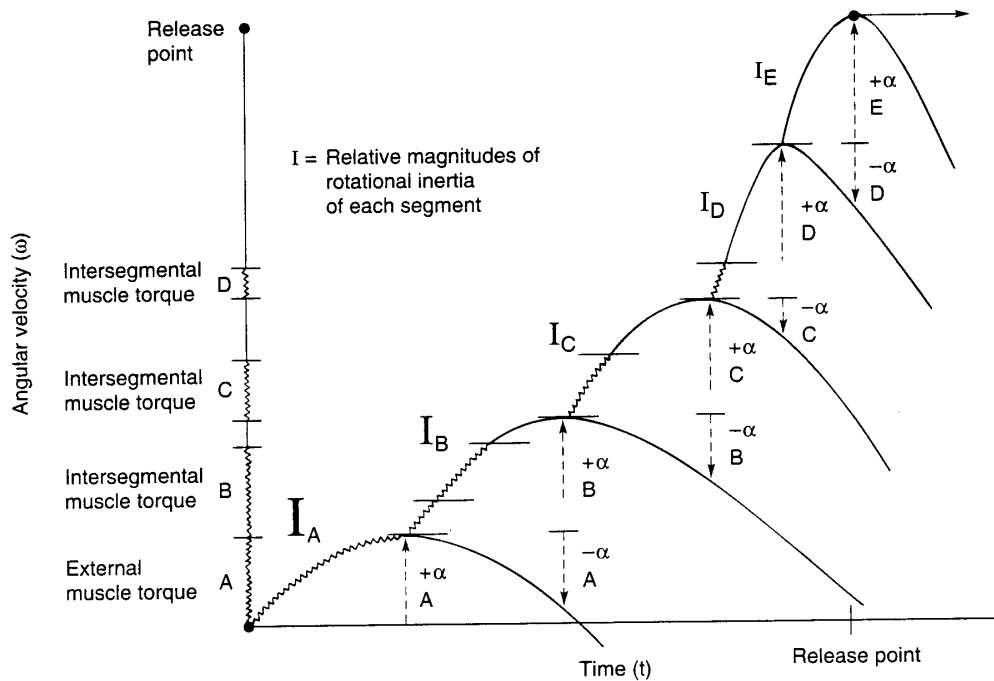
- Ideally deceleration of proximal segments occurs soon after mid range when $\omega = \text{maximum}$ for proximal segment.

The deceleration of a proximal segment maybe caused by:

1. Muscles that are antagonists to the motion occurring in the proximal segment.
2. Inter-segmental muscles between the proximal segment and the next distal segment – As the adjoining distal segment accelerates the proximal segment decelerates.

4.2.1 A Model of an Open Kinetic Link System.

To illustrate the effect of acceleration and deceleration of proximal segment on distal ones, consider the following conceptual model.



A conceptual model of the kinetic link principle.

This is a five segment system and may be truncated or added to.

A, B, C, D, E; proximal \rightarrow distal

Example:

- A – pelvis rotating about longitudinal hip axis
- B – trunk rotating about longitudinal vertebral Axis
- C – shoulder girdle protracting about longitudinal sternoclavicular axis

D – arm	medially rotating about longitudinal shoulder axis
E – forearm	extending about an ML elbow axis
F – hand	hand (and ball) flexing about ML wrist axis

- Muscle torques denoted by ‘squiggly’ lines.
- Shows ω as a function of time (ω – angular velocity of end segment)
- $+\alpha$ - accelerating segment
- $-\alpha$ - decelerating segment

I – moment of inertia. The I 's decrease in size to signify the decrease in I as we go proximal to distal.

ω still continues to rise even when muscle torques stop because of the decreasing I as the proximal segment decelerates.

Throw-like movements performed while in the air?

4.3 Lever vs. Wheel-Axle Rotations.

- Lever and wheel-axle mechanisms, were discussed in Biomechanics 1.
- Both are used in segmental movements associated with the kinetic link model.

Lever-type motions are most common (flexion-extension, abduction-adduction, protraction-retraction)

Wheel-axle motions occur around longitudinal axis of segment (medial and lateral rotation, left and right rotation, pronation-supination, inversion-eversion)

- Both are rotating systems \Rightarrow if a high linear velocity is desired then both ω and r (radius of rotation) should be maximized ($v = r\omega$).

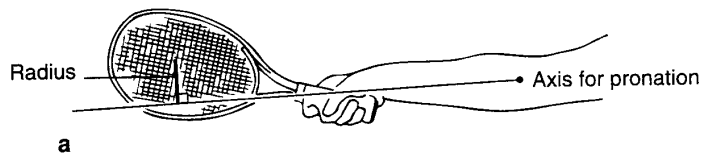
Consider r :

In the throwing, kicking and striking movement one often likes to adjust the distance from point of contact (or release) to axis of rotation, i.e. r .

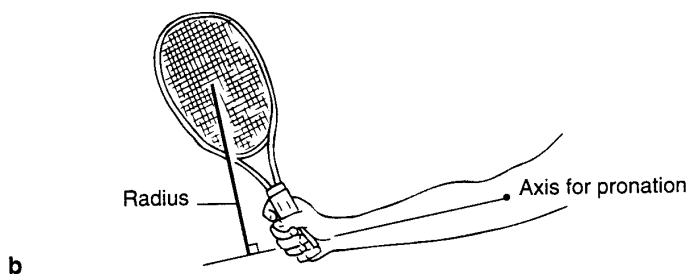
Example:

In a kicking skill using knee extension, the leg and foot acts as the lever – to decrease the r for this lever-like movement, one must contact the ball on the tibia – an unlikely choice.

- Unlike lever systems, the wheel-axle mechanism has an adjustable r . By extending the ‘wheel’ segment relative to the axle segment r can be increased or decreased.



(a) Small-radius “wheel” segment.
(b) Large-radius “wheel” segment, used in the wheel–axle motion of radioulnar pronation.



- Never the less levers tend to have the larger r
– because they use the ‘full’ extension of segments.

Consider ω :

The amount of ω a segment achieves depends upon I and

τ .

$$\alpha = \frac{\tau}{I}$$

$$\text{or } \alpha = \frac{\tau}{mk^2}$$

$$\text{and } \omega \sim \alpha t$$

For a fixed τ , α can be changed only by changing k . So,

Smaller $k \Rightarrow$ larger α

or

larger $k \Rightarrow$ smaller α

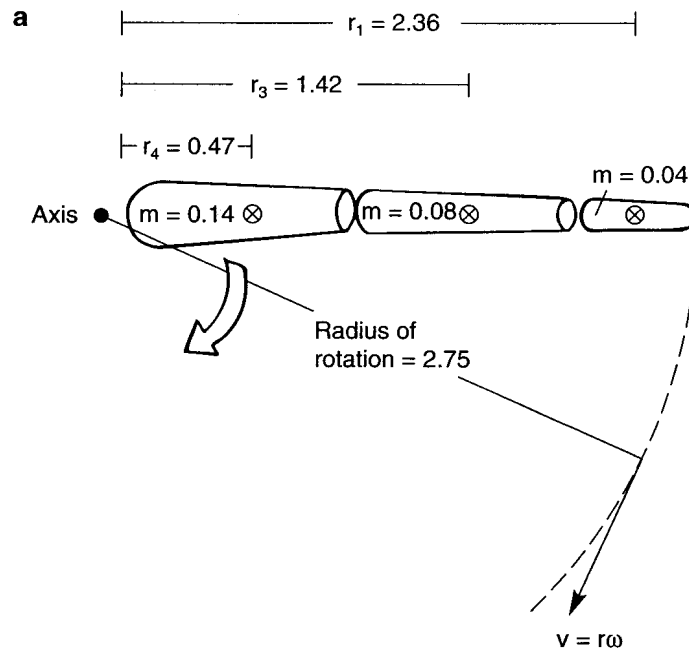
This puts us in a dilemma!

If one increases r to increase v ($v = r\omega$) one also increases

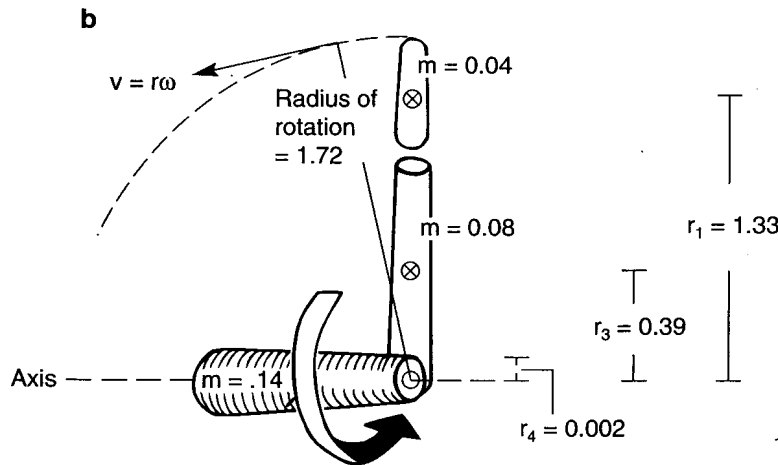
$k \rightarrow$ increase in $I \rightarrow$ decrease in $\alpha \rightarrow$ decrease in ω .

There is a way to theoretically solve this problem.

Consider the following lever and wheel-axle arrangements.



$$\begin{aligned}
 I_L &= \sum mr^2 + I_{cg} \\
 &= (0.14)(0.47)^2 + 0.0157 = 0.05 \\
 &= (0.08)(1.42)^2 + 0.0156 = 0.17 \\
 &= (0.04)(2.36)^2 + 0.0004 = 0.22 \\
 \hline
 I_L &= 0.44 \text{ s-ft}^2
 \end{aligned}$$



$$\begin{aligned}
 I_{WA} &= \sum mr^2 + I_{cg} \\
 &= (0.14)(0.002)^2 + 0 = 0 \\
 &= (0.08)(0.39)^2 + 0.0056 = 0.02 \\
 &= (0.04)(1.33)^2 + 0.0004 = 0.07 \\
 \hline
 I_{WA} &= 0.09 \text{ s-ft}^2
 \end{aligned}$$

$$\frac{\text{Lever } I_A}{W - A I_A} = \frac{0.44}{0.09} \approx \frac{5}{1}$$

$$\frac{\text{Radius of rotation lever}}{W - A \text{ radius of rotation}} = \frac{2.75}{1.72} \approx \frac{1.6}{1} \approx \frac{2}{1}$$

$$I_z = \frac{1}{2} mr^2$$

$$\begin{aligned}
 V_L = r\omega &= \frac{2}{1} \frac{1}{5} = \frac{2}{5} \\
 V_{WA} = r\omega &= \frac{1}{2} \frac{5}{1} = \frac{5}{2}
 \end{aligned}$$

The rotational inertia of a segment in (a) a lever arrangement and (b) in a wheel-axle arrangement. I_{cg} designates the rotational inertia of a segment about its own center of gravity.

Therefore if $\omega_L = \omega_{WA}$ a much greater muscle torque will be required to produce ω_L , than ω_{WA} .

‘Fairer’ to start with equal τ ’s.

$$\alpha = \frac{\tau}{I} \quad \text{and} \quad \omega \sim \alpha t \quad \text{so} \quad \omega \sim \frac{\tau}{I} t$$

If the same τ is applied for the same t ($\tau_L = \tau_{WA}$) then;

$$\omega_{WA} = 5\omega_L \quad \text{i.e.,} \quad \frac{\omega_L}{\omega_{WA}} = \frac{1}{5}$$

Thus,

$$\begin{aligned} \frac{v_L}{v_{WA}} &= \frac{r_L \omega_L}{r_{WA} \omega_{WA}} = \frac{r_L}{r_{WA}} \times \frac{\omega_L}{\omega_{WA}} \\ &= \frac{2}{1} \times \frac{1}{5} = \frac{2}{5} \end{aligned}$$

or

$$v_{WA} = \frac{5}{2} v_L$$

\Rightarrow Wheel-axle will give $2\frac{1}{2}$ times the end-point velocity of the lever (theoretically).

Note: the assumption of $\tau_L = \tau_{WA}$ is questionable. The τ -producing capabilities of the muscle groups used should be considered.

Summary:

Levers – greater potential for high v because of larger r .

Wheel-axle – greater potential for high v because of small I .

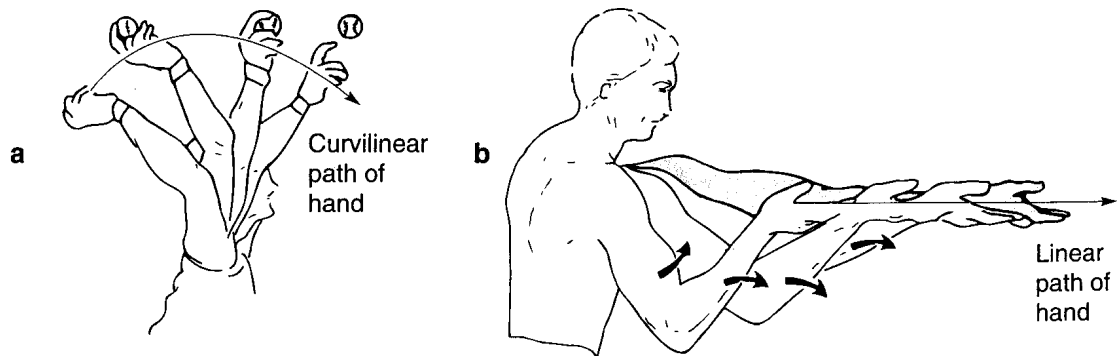
The body uses both depending upon circumstances.

4.4 Push-like Patterns.

- In the previous section we looked at throw-like motion. The aim was to produce high end-point velocity – sequential segmental rotations.
- These movements include throwing, kicking and striking skills.
- For other skills the performance objectives may be:
 1. Accuracy of projection, e.g. basketball throw.
 2. Application of a large motive force to overcome the resistive force acting on an object, e.g. weight lifting, body projecting itself.

Throw-like pattern → high v → point of contact of object on distal link moves in rotary or curvilinear path.

Push-like pattern → accuracy or overcoming resistive force → point of contact with object moves in rectilinear path.



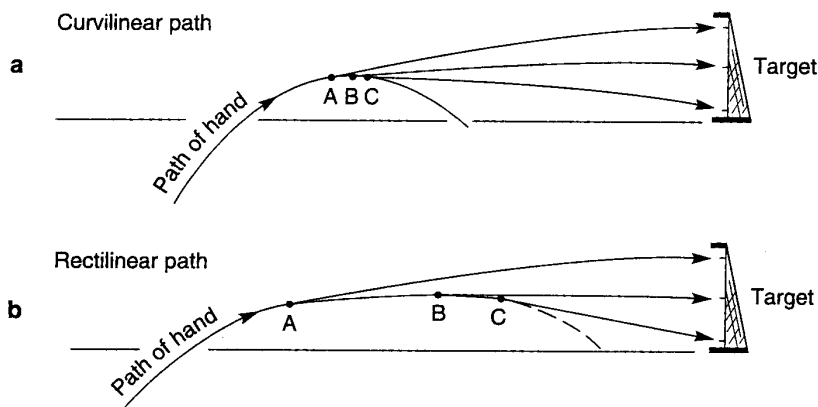
(a) An overarm throw. (b) A fencing lunge.

In push-like patterns the segmental rotations take place simultaneously, rather than sequentially.

– This allows control so that the path is a straight line.

Curve path \Rightarrow object must be released (or struck) within a narrow space to hit a target – little time is available within the total performance to contact or release the object and be successful.

Straight path \Rightarrow object can be released or struck within a wide range and still hit target – performer has more time to align the release or contact to be successful.



The above figure shows the difference in accuracy of projection between a curvilinear path and a rectilinear path.

Example:

Students in a beginning tennis class tend to ‘push’ the ball rather than strike it in order to achieve an accurate serve.

4.4.1 Differences between Throw-like and Push-like patterns.

1. Throw-like – segment-object contact point ‘lags back’ as the proximal segments ‘move out from under’ the distal segment and eventually the distal end will catch up to (and maybe pass) the proximal segments at release.

Push-like – segments are positioned either behind the object to be projected (darts) or the object or implement is pulled along (oar in rowing, ball and stick in field hockey push-in)

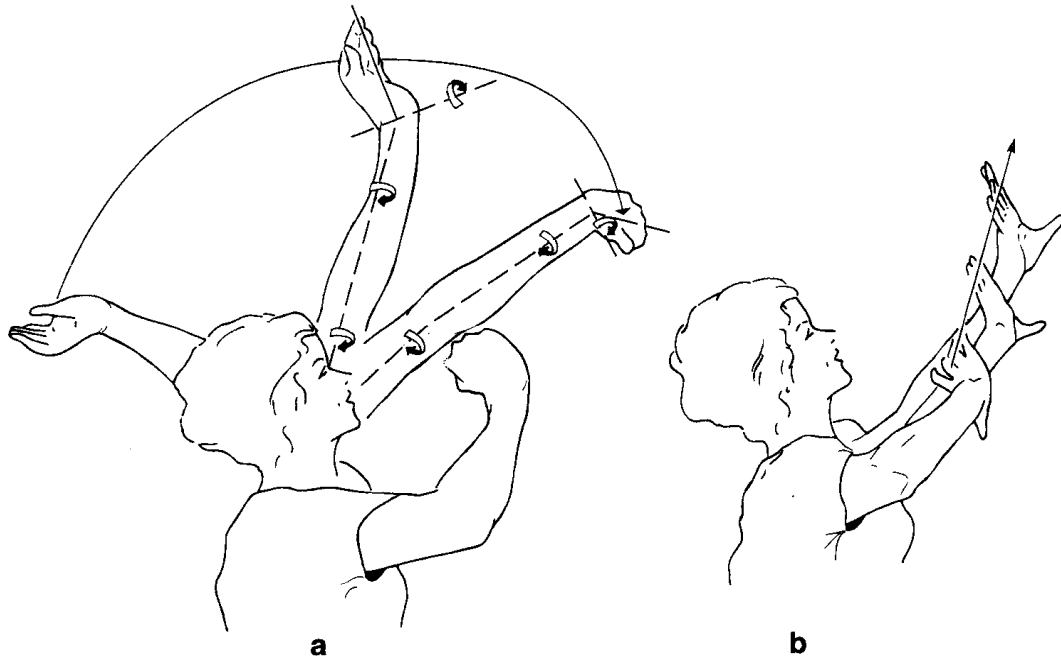
2. Throw-like – segmental rotations occur sequentially to produce high velocity.
Push-like – segmental rotations occur simultaneously to produce high accuracy and/or force.

3. Throw-like – object moves along a curvilinear path before contact or release.
Push-like – object moves along a rectilinear path before contact or release.

4. Throw-like – predominance of wheel-axle movements.
Push-like – predominance of lever-like movements.

Example:

Compare the volleyball spike and volleyball set pattern.



(a) Curvilinear motion and (b) rectilinear motion caused by the spatial and temporal characteristics of the segmental movements.

Spike:

- Aim – high velocity.
 - Motion of hand is curvilinear
 - Initial lag of hand
 - There is sequential rotations
- Typical throw-like

Set:

- Aim – accurate placement
 - Motion of hand is rectilinear
 - Hands start in front
 - There is simultaneous rotations
- Typical push-like

A push-like pattern also is used in situations in which the centre of gravity of an object being moved must be kept in a precise location for balance.

Example:

In lifting free weights in weight training, the centre of gravity of the barbell must be kept in vertical alignment with the base segments. If the barbell moves horizontally from its vertical rectilinear path, other muscle groups must be used to bring it back into alignment, or it will rotate over and fall to the floor.

Weight-lifting machine eliminates the need for the performer to maintain the vertical rectilinear direction of the resistance → easier and safer lifting!

Whether a skill ideally should be a throw-like or a push-like motion depends primarily on the overall performance objective of the skill.

– High velocity

or

– Accuracy/manipulating large resistance.

The choice of pattern also depends on the constraints of the activity and the performer.

The ideal pattern may have to be compromised due to the physical environment, competitive situation or physical attributes of the performer.

4.5 The Throw-Push Continuum.

- Open kinetic link principle applied to the throw-like pattern governs events demanding high velocity
- Open kinetic link principle applied to the push-like pattern governs events demanding accuracy, guiding objects in an accurate path or moving relatively large resistances.
- In general, open kinetic chain movements cannot be classified as being entirely throw-like or entirely push-like.

- Constraints in activities usually require ‘blending’ of throw-like and push-like patterns into combinations of the two.
- For this purpose, skills can be placed along a throw-push continuum.
 - Skills performed with entirely sequential rotations are placed at one end.
 - Skills performed with entirely simultaneous rotations are placed at the other end.
 - In between are skills that display both sequential and simultaneous rotations.

Throw-like patterns are characterized by the sequencing of segmental rotations from most massive to least massive and from most stable to most free.

The constraints that influence the location of a skill on the continuum are as follows:

1. The massiveness of the object to be moved or projected.
2. The size of the object to be moved or projected.
3. The shape of the object.
4. The strength of the performer.
5. The skill of the performer.

Example:

If high speed of the object is important for achieving distance of projection – then the most sequential pattern possible should be used.

For a skilled player to throw a cricket ball from the outfield – using sequential rotations entirely is both possible and desirable.

In the shot put, high speed is still the most important parameter for achieving distance of projection – performance objective is as above.

However, the shot would not be thrown because of distal musculoskeletal limitations in manipulating a massive object at its end-point.

One should attempt to produce sequential rotations of the massive segments (pelvis, trunk and shoulder girdle) followed by simultaneous rotations of the less massive distal segments of the upper extremity.

⇒ shot put ~ in the middle of continuum.

~ ½ sequential and ½ simultaneous.

It is an activity in which constraints of the performer demand the use of less than the ideal pattern.

In the case of the weak performer attempting to put the shot further adaptation is necessary – all simultaneous motions may be used, without any evidence of sequencing.

⇒ an activity that should ideally be ‘pure’ throw-like ends up ‘pure’ push-like.

Size and Shape (second and third constraints):

A javelin and football are also thrown for distance but because of the somewhat unwieldy shapes there is a small component of a push at the distal end.

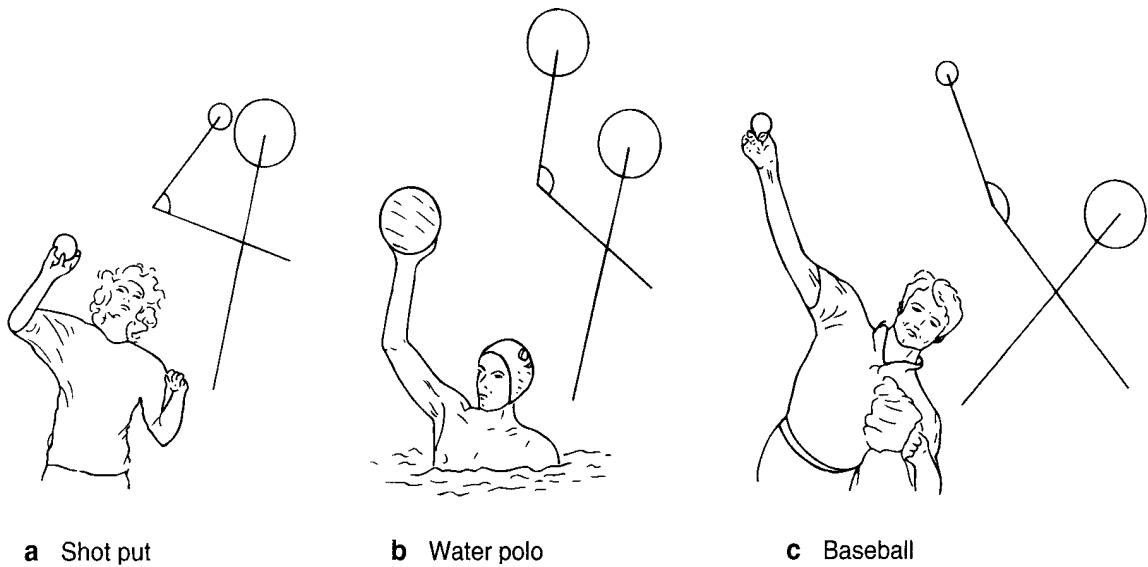
Javelin is thrown in the manner because of tip placement.

Football is thrown in this manner because of possible wobble.

One identifying characteristic of a throwing pattern that has been adapted to a partial pushing pattern is that the object to be projected is brought in closer to the longitudinal body axis.

This is done because I produced by the distal segment and object is too great for smaller muscle torques to stabilize or accelerate.

As the mass is brought closer to the body, the sequential segmental rotations become more simultaneous segmental rotations and combine to move the distal end of the kinetic chain in a rectilinear rather than a curvilinear path before release.



Relative positions of body segments and objects for three different masses and sizes of objects being projected.

4.6 Performance Analysis of Push-like Movements,

There are three mechanical purposes related to push-like patterns:

1. To manipulate a resistance – force activities
2. To generate maximum power – power activities
3. To maximize accuracy – accuracy activities.

4.6.1 Force Activities.

In force activities the performer aims to move a resistance from A to B. How fast this is done is not significant to the outcome.

‘Power lifting’ is a good example of this. In reality the power lifting events of bench press, squat and dead lift are force activities not power activities.

Maximum strength is paramount not the speed of the lifts. Brown and Aboni (1985) found that skilled lifters took a longer time to perform the dead lift than the unskilled.

→ Force-velocity relationship of muscle.

To achieve maximum force against the resistance and straight-line motion, simultaneous segmental rotations are necessary.

Squat:

Slight extension of inter-vertebral joints and extension of hips and knees and plantar flexion of ankles.

All movements should occur simultaneously to maximize muscle torque against resistive torque and to keep bar moving vertically up.

Accelerations should be kept to a minimum – injury prevention and force-velocity relationship.

Skilled lifters show less variation in speed of both descent and ascent than unskilled lifters.

4.5.2 Power Activities.

Power activities, like force activities require a high amount of force.

In addition however, power activities require a high velocity.

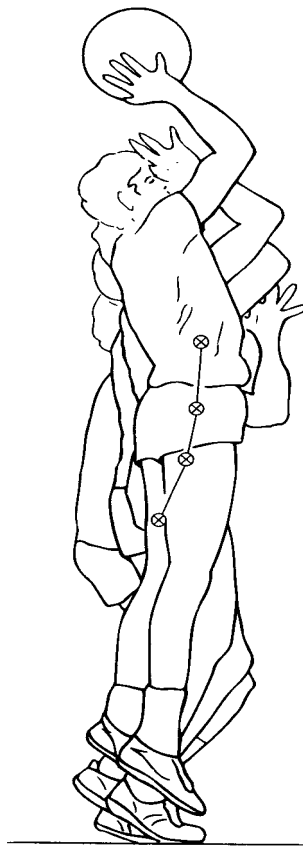
Force-velocity relationship \Rightarrow when v increases, F decreases and vice versa.

\Rightarrow percentage of maximum muscle tension required to lift a given load increases with speed of lift.

In power activities the velocity of the object being moved must be established in a relatively short time \Rightarrow high acceleration.

Limited time available for force development.

- Jumping events include jumps that are discrete skills in themselves and jumps that are part of a larger skill, e.g. volleyball spike, basketball jump shot, takeoffs in gymnastics and diving.
- Whenever the body is projecting itself, the segmental rotations act to move the body's centre of gravity in a rectilinear path.



Segmental rotations producing a linear path of the center of gravity.

- In contrast to most other events, the jumping event shows the most massive segment, the trunk, located at the open end of the link system and the least massive, the feet, located at the fixed end.
- Segmental movements are in reverse order.

most free —————→ most fixed (feet)

- To project the body one must have an external force – this is the ground reaction force (F_R) – created by the jumper pushing against it.
- F_N is applied to feet and should ideally pass through the body's centre of gravity. If F_N passes in front or behind the centre of gravity → a rotation moment → loss of muscle work and height.
- The motion consists of a sequential application of musculotendinous power from the hip to the ankle.
- The initial upward motion of the sequence is arm flexion.

upward arm acceleration → arms exerting a downward reaction force on the segments below → preloading of lower segments (storage of elastic recoil energy)

Deceleration of arms → unloads lower segments ⇒ optimum time to begin trunk extension.

- Ideal push motion has simultaneous rotation of segments.

However in jumping the lower and less massive segments are subject to a large downward reaction force during upward acceleration of the trunk.

⇒ They may be unable to begin simultaneous extension with the trunk, even though their muscles are attempting to produce the force for upward acceleration.

Advantage?

Additional pre-stretching of lower extensor muscles

→ evoke stretch reflex and increase elastic strain energy.

Punching motions:

- Boxing, karate, tae kwon do, etc.
- Both force and speed of punch are important.

Boxing:

A right hand punch is performed most effectively by using hips, trunk and shoulder girdle segments as a wheel-axle mechanism about the supporting opposite hip joint. The right upper extremity is used as a lever system – shoulder flexion and elbow extension.

Straight-line movement of fist is important

→ shortest distance between two points ⇒ faster punch.

Jab – hook?

Acceleration of hand (fist) is very important. The success of the impact is determined to a large extent by the sudden deceleration of the hand on the opponents face.

– A larger acceleration → a greater impact velocity → a greater deceleration → a larger impact force.

The effectiveness of the punch relies on both impact power and accuracy – rectilinear best for accuracy.

The peak forces produced on impact do most of the damage.

An opponent can reduce these peak forces by ‘rolling’ with the punch.

Tae kwon do:

Turning hook kick is an example of a punching movement, while rotating on one leg. The lower extremity of the other leg undergoes extension (push) at the end of the rotation to contact the target

- Horizontal abduction of thigh is followed by hip, knee and ankle extension, so the foot arrives at the target as quickly as possible.
- Like boxing contact speed is important – same reasons.

Lifting events:

Olympic lifts – snatch and clean and jerk – have a high component of power.

Both lifts require the bar to be moved upward as fast as possible in order for the performer to have sufficient time to ‘drop’ under the bar.

In the clean and jerk this is done twice – once in the clean and again in the jerk.

In the snatch it is done once.

The faster the bar moves up, the more time the performer has to drop under and less hip and knee flexion is required to do this.

– ‘Recovery’ after dropping under is a force activity.

Continuous power events:

Whenever there is limited time to accelerate an object, power is involved.

Continuous power events include activities that require the repeated application of a force under time constraints.

Two examples are rowing and sprinting.

Rowing:

Rowing motion involves two blocks of segmental movements:

1. Extension of trunk, hips, knees and ankles.
2. Retraction of shoulder girdle, extension-horizontal abduction of shoulder and elbow flexion.

Movement of segments in the blocks tends to be simultaneous, while the block themselves are sequenced.

After start up period, very little if any slippage occurs between oar and water.

- An oarsperson must apply a large force in a limited time to keep the shell moving at high velocity or accelerating.
- The oar moves at the same speed as the shell (after the start)
 - ⇒ The oarsperson must produce enough force by contracting the muscles fast enough to surpass hydrodynamic drag forces and apply additional force if possible.
- Start-up is force dominated power event, rest is speed-dominated power event.

Sprint:

Same principle as rowing

After a force dominated power start, the runner must generate sufficient force, during the brief time the foot is in contact with the ground, by rapid contract of muscle to support the body weight and keep the body moving at a high speed against aerodynamic drag forces.

Power surge comes during the short time the foot is in contact with the ground ($\rightarrow F_{Rx}$ & F_{Ry})

Speed of running is strongly correlated with stride length and stride frequency.

A greater power in push-off \rightarrow greater stride length and the faster the runner can get the foot back down to the ground (i.e. the greater the stride frequency) the more frequently the power can be applied.

Push-like simultaneous extensions of the hip, knee and ankle joints provide the power in this skill.