SP3005/PH3025 ADVANCED BIOMECHANICS
Laboratory 1
Musculoskeletal Machines - Levers

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14. September, 2004

Contents

1 Introduction

The most common machine found in the human musculoskeletal system is the lever. A simple lever system consists of four componenets:

1. A rigid lever
2. An axis of rotation or fulcrum
3. A motive force
4. A resistive force.

In the body the long bones act as the levers, the axes of rotation are located in the joints, the muscles supply the motive force, and external loads produce the risistive force.

2 Theory

Lever are divided into three classes, depending upon the relative positions of the axis, the motive force and the resistive force.

First-class lever systems The axis of rotation is located between the motive force and resistive force.

Second-class lever systems The resistive force is located beween the motive force and the axis of rotation.

Third-class lever systems The motive force is located between the risistive force and the axis of rotation.

Machines are used because they make work "easier", or more specially they give the user a **mechanical advantage** (MA). This mechanical advantage may be in force, range of motion (ROM), or speed of movement. In terms of force mechanical advantage is defined as:

$$MA = \frac{resistive\ force}{motive\ force} = \frac{motive\ force\ arm}{resistive\ force\ arm}$$

With respect to force the musculoskeletal system is designed for a mechanical disadvantage, i.e., the motive force generated by the muscles must be greater than the resistive force of the external load in order to overcome it. However the musculoskeletal system does have a mechanical advantage in ROM and speed of movement. The mechanical advantage in ROM and speed of movement is defined to be the reciprocal of equation (1). Thus if a particular musculoskeletal lever has a mechanical advantage of 0.1 in force (or a mechanical disadvantage of 10 in force) then it will have a mechanical advantage of 10 in ROM and speed of movement.

3 Aims

The aims of the present practical are to:

1. become familiar with lever systems
2. make measurements of resistive forces and motive forces
3. calculate MA's
4. Compare measured and calculated values of the motive force
5. Calculate the joint reaction force.

4 Equipment

- Lever system
- Weights
- Load cell
- Goniometer

5 Procedure

Determine the weight and centre of mass of the lever. Calibrate and zero the load cell.

Part A

Most musculoskeletal lever systems in the body are third class. Set the lever system up for a third-class lever:

(i)
- Select an external load, an "origin" and an "insertion".
- Select the origin high and the insertion close to the joint.
- Weigh the load and measure the appropriate angles and distances. Be sure to state the associated errors. From these measurements calculate the "muscle" tension.
- Now measure the muscle tension with the load cells, how does it compare to the calculated value?
- Calculate the MA of the system
- What is the mechanical disadvantage in force?
- What is the mechanical advantage in ROM and speed of movement?

(ii) For the same load and origin as in (i), move the insertion further away from the joint.

- Measure the muscle tension.
- Calculate the rotary and the stabilizing component of the muscle tension.
- Calculate the MA and compare with that from (i)
- What advantage do people with muscle insertion further away from a joint have over those with muscle insertion close to a joint?

(iii) For the same load and insertion as in (ii), move the origin closer to the joint.

- Repeat the measurements made in (ii), compare with the present and comment
- Explain why the MA is less.
- Calculate and measure the motive force arm.

(iv) For one of the arrangements above calculate the joint reaction force.

Part B

Construct a second-class lever system and determine its MA. Are there any second-class levers in the human body? Explain.

Part C

With the present apparatus, can you construct a first class lever system?

6 Results and Discussion

The description of the values used to determine the results of the measurements can be found in the appendix (section A). There are also all the equations which will be used to derive the calculated results.

To be able to derive the mass of the lever the following data was measured:

$$\begin{aligned} \beta &= 60^\circ \\ \phi &= 66^\circ \\ l_{insertion} &= 0.52\ m \\ l_{lever} &= 0.58\ m \\ F_{muscle} &= 1.55 * 9.81\ N = 15.21\ N \end{aligned}$$

With equation 6 F_{weight} is calculated for $\underline{F_{weight} = 28.76\ N}$ ($m_{lever} = 2.9\ kg$) at $\underline{l_{com} = 0.29\ m}$.

point	l_{iload} (m)	$F_{iweight}$ (N)
1	0.58	14.39
2	0.38	21.96
3	0.18	46.35

Table 1: Additional weight to apply on the external load points (eqn. 7)

The external load for all following results was chosen with 8 kg ($F_{load} = 9.81 * 8\ N = 78.48\ N$).

Part A

(i) Terms of measurement:

$$\begin{aligned} \beta &= 86^\circ \\ \phi &= 81^\circ \\ l_{insertion} &= 0.07\ m \\ l_{origin} &= 0.60\ m \end{aligned}$$

The mechanical MA of the system can be found in table 2. A muscle can lift much more weight if that is placed nearer to the joint due to the

point	measured F_{muscle} (N)	l_{load} (m)	$F_{iweight}$ + F_{load} (N)	MA_F	MA_l	calculated (eq. 8) F_{muscle} (N)
1	65.63*9.81 = 643.83	0.58	92.87	0.14	0.12	777.19
2	47.10*9.81 = 462.05	0.38	100.44	0.22	0.18	550.70
3	27.55*9.81 = 270.27	0.18	124.83	0.46	0.39	324.20

Table 2: Results for (i)

$$MA_F = \frac{(F_{iweight} + F_{load}) sin(\beta)}{F_{musle} \sin(\phi)} \qquad MA_l = \frac{l_{insertion}}{l_{load}}$$

decrease of the mechanical disadvantage and the increase of the MA.

But the muscle only has to move a small distance on the insertion point to achieve a big change on the load point. This is the mechanical advantage in ROM. And it is also an advantage in speed because a bigger distance can be covered in a short time.

(ii) Terms of measurement:

$$\begin{aligned} \beta &= 75° \\ \phi &= 80° \\ l_{insertion} &= 0.22\ m \\ l_{origin} &= 0.60\ m \end{aligned}$$

point	measured F_{muscle} (N)	l_{load} (m)	$F_{iweight}$ + F_{load} (N)	MA_F	MA_l	calculated (eq. 8) F_{muscle} (N)
1	22.34*9.81 = 219.16	0.58	92.87	0.42	0.38	240.14
2	15.94*9.81 = 156.37	0.38	100.44	0.63	0.58	170.16
3	9.40*9.81 = 92.21	0.18	124.83	1.33	1.22	100.18

Table 3: Results (ii)

$$MA_F = \frac{(F_{iweight} + F_{load}) sin(\beta)}{F_{musle} \sin(\phi)} \qquad MA_l = \frac{l_{insertion}}{l_{load}}$$

The mechanical advantage (MA) for the same external load on the same load point was much greater this time (table 3) than measured in (i) (table 2). The reason for it can be found in the length of the motive force arm (postion of the insertion point farther away from the joint). Because of its greater length more of the muscle force takes effect on the point where the load acts.

point	F_{muscle} (N)	rotary $F_{Vmuscle}$ (N)	stabilizing $F_{Amuscle}$ (N)
1	219.16	215.83	37.48
2	156.37	153.99	27.15
3	92.21	90.81	16.01

Table 4: Components of the muscle tension (eqn. 3)

As shown in table 4 more than 82% of the muscle tension is used to produce a moving torque. Therefore, people with muscle insertion further away from a joint have the advantage to be able to lift greater weights as equally trained people with insertion close to a joint. But, on the other hand, the later mentioned people are faster in movement so they should show a better result in reaction tests.

(iii) Terms of measurement:

$$\begin{aligned} l_{insertion} &= 0.22\ m \\ l_{load} &= 0.58\ m \\ F_{iweight} + F_{load} &= 92.87\ N \end{aligned}$$

point	measured F_{muscle} (N)	l_{origin} (m)	ϕ (°)	β (°)	MA_F	calculated (eqn. 9) $l_{insertion}$ (m)
1	22.40*9.81 = 219.74	0.60	81	75	0.41	0.24
2	27.05*9.81 = 265.36	0.45	43	115	0.47	0.27
3	28.28*9.81 = 277.43	0.30	43	95	0.49	0.28

Table 5: Results (iii)

$$MA_F = \frac{(F_{iweight} + F_{load}) sin(\beta)}{F_{musle} \sin(\phi)}$$

$$MA_l = \frac{l_{insertion}}{l_{load}} \rightarrow \underline{MA_l = 0.38}$$

The calculated value of $l_{insertion}$ derived from the before measured values (table 5) (the length of the motive force arms) does deviate from the real length of the motive force arms. This variance partly occurs because of the unprecise way to meter the angles.

The MA is less because of the smaller distance between join and origin. The insertion angle flattenes and therefore the share of the force which acts as a torque to move the lever becomes smaller. On the other hand the stabilizing component of the force is increased.

(iv) With equation 10, 11 and the values from the first line of table 5 it is possible to calculate the joint reaction force:

$$\begin{aligned}\beta &= 75^\circ\\ \phi &= 81^\circ\\ F_{iweight} + F_{load} &= 92.87\ N\\ F_{muscle} &= 219.74\ N\end{aligned}$$

$$\underline{F_{Xjoint} = -89.4\ N} \qquad \underline{F_{Yjoint} = -107.87\ N}$$

Part B

Measured data:

$$\begin{aligned}\beta &= 60^\circ\\ \phi &= 66^\circ\\ l_{insertion} &= 0.52\ m\\ l_{load} &= 0.38\ m\\ m_{weight} &= 8\ kg\\ F_{muscle} &= 7.09 * 9.81\ N = 69.55\ N\\ F_{iweight} + F_{load} &= 100.44\ N\end{aligned}$$

The calculated value (equation 8) of $F_{muscle} = 69.58\ N$ is nearly the same as the measured one.

$$MA_l = \frac{l_{insertion}}{l_{load}}$$

The system has an MA of 1.37.

In the second-class lever system the resistive force is located between the motive force and the axis of rotation. A smaller effort can be used to advantage over a larger weight. In the human body a second-class lever is, for example, the Achilles tendon, pushing or pulling across the heel of the foot.

Part C

It is not possible to construct a first class lever system with the present apparatus.

7 Conclusion

The practical shows very clear that to produce a sufficient torque not only the size of force is crucial. But also the point were the force works on the lever is very important. It illustrates how artful the human body is constructed by nature with respect to the basic laws of physics in mind. Every combination of lever and muscle is precisely designed and optimized for its special purpose.

The effect which will be produced by a slight change of the angle can be easily underestimated and therefore needs to be calculated, for the specific case, very carefully in a mathematic model. Even small deviations during measurement can cause quite significant differences between the measured and the expected values as seen in the laboratory, too.

A Calculation

A.1 Assumptions

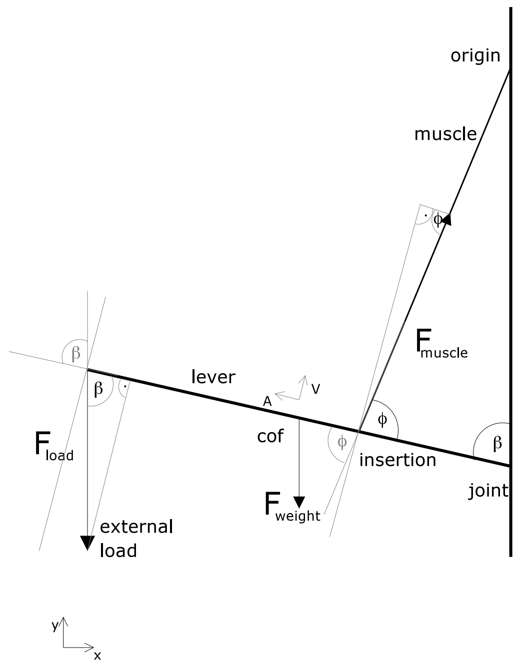

Figure 1: sketch

The lever system is sketched in figure 1. To keep clearness there are no length inscribed in this sketch.

l_{origin} the length from the joint to the origin

l_{load} the length from the joint to the load injection point

$l_{insertion}$ the length from the joint to the insertion point

l_{com} the length from the joint to the center of mass of the lever

l_{lever} the length from the joint to the end of the lever

Furthermore, there are no angles marked on the sketch for F_{weight} because the calculation is analog to F_{load}. Forces which take effect along the lever (stabilizing component) will be called F_{Ai} and vertical (rotary component) ones F_{Vi} (with i=weight, load, muscle).

$$F_{V load} = F_{load} sin(\beta) \qquad F_{A load} = F_{load} cos(\beta) \tag{1}$$
$$F_{V weight} = F_{weight} sin(\beta) \qquad F_{A weight} = F_{weight} cos(\beta) \tag{2}$$
$$F_{V muscle} = F_{muscle} sin(\phi) \qquad F_{A muscle} = F_{muscle} cos(\phi) \tag{3}$$

A.2 Motive force

$$\begin{aligned} join\circlearrowleft : \quad 0 &= l_{com} F_{V weight} + l_{load} F_{V load} - l_{insertion} F_{V muscle} \\ l_{insertion} F_{V muscle} &= l_{com} F_{V weight} + l_{load} F_{V load} \\ F_{V muscle} &= \frac{l_{com} F_{V weight} + l_{load} F_{V load}}{l_{insertion}} \end{aligned}$$

$$F_{muscle} = \frac{(l_{com} F_{weight} + l_{load} F_{load}) sin(\beta)}{l_{insertion} sin(\phi)} \tag{4}$$
$$l_{insertion} = \frac{(l_{com} F_{weight} + l_{load} F_{load}) sin(\beta)}{F_{muscle} sin(\phi)} \tag{5}$$

A.3 Lever weight and centre of mass

The lever model is just a simple bar. Therefore the center of mass will be in the middle of the lever ($l_{com} = \frac{l_{lever}}{2}$). The easiest way to measure the weight of the lever is to measure just the ”muscle tension” without any external load.

$$0 = l_{com} F_{V weight} - l_{insertion} F_{V muscle}$$

$$\frac{l_{insertion} F_{muscle} sin(\phi)}{l_{com} sin(\beta)} = F_{weight}$$

$$\frac{2 l_{insertion} F_{muscle} sin(\phi)}{l_{lever} sin(\beta)} = F_{weight} \tag{6}$$

Therefore, it is possible to assume the lever as a weightless bar and to add the appropriate force to the load force. In order to do this a replacement weight force must be calculated for every point where the external load is connected ($F_{i weight}$ with i=1, 2, 3) and can than be added to the load force.

$$\frac{l_{insertion} F_{muscle} sin(\phi)}{l_{iload} sin(\beta)} = F_{iweight} \tag{7}$$

with $F_{iweight}$ equation 4 and 5 become to:

$$F_{muscle} = \frac{(F_{iweight} + F_{load}) l_{load} sin(\beta)}{l_{insertion} sin(\phi)} \tag{8}$$

$$l_{insertion} = \frac{(F_{iweight} + F_{load}) l_{load} sin(\beta)}{F_{muscle} sin(\phi)} \tag{9}$$

A.4 Joint reaction force

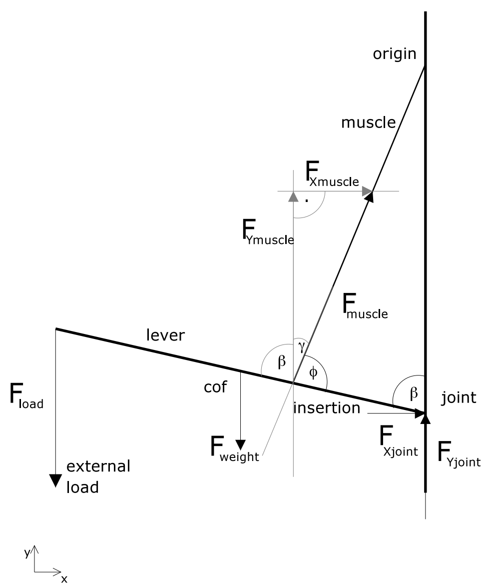

Figure 2: Joint forces

A specific sketch more suitable to calculate the joint forces is shown in figure 2.

$$\gamma = 180° - \beta - \phi$$

$$
\begin{aligned}
\rightarrow: \qquad 0 &= F_{Xjoint} + F_{Xmuscle} \\
F_{Xjoint} &= -F_{Xmuscle} \\
F_{Xjoint} &= -F_{muscle} sin(\gamma)
\end{aligned}
$$

$$
\begin{aligned}
\uparrow: \qquad 0 &= -F_{load} - F_{weight} + F_{Ymuscle} + F_{Yjoint} \\
F_{Yjoint} &= F_{load} + F_{weight} - F_{Ymuscle} \\
F_{Yjoint} &= F_{load} + F_{weight} - F_{muscle} cos(\gamma)
\end{aligned}
$$

$$F_{Xjoint} = -F_{muscle} sin(180^\circ - \beta - \phi.) \tag{10}$$

$$F_{Yjoint} = F_{load} + F_{weight} - F_{muscle} cos(180^\circ - \beta - \phi) \tag{11}$$